Internet Appendix

A Visit to the Spectral Zoo

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Section 1 provides a brief review of filters. Section 2 explains why beta-decompositions based on covariances of the form Cov $\left(\widetilde{D}_{j,t}^{f}, \widetilde{D}_{j,t}^{R_i^e}\right)$ are not properly defined. Section 3 describes our frequency-specific data generating process. Section 4 discusses the bootstrap approach that we use in the main paper and the rank tests that we report here. Section 5 contains additional results and robustness checks that are omitted in the main paper for brevity.

1 Notes on Filters

1.1 Basic Definitions

Our review follows Percival and Walden (2000). We also recommend the excellent analysis of filter banks by Strang and Nguyen (1996).

A filter $\{h_l : l = 0, \dots, L-1\}$ of even width L (implying $h_0 \neq 0$ and $h_{L-1} \neq 0$) is called a wavelet filter if $\sum_{l=0}^{L-1} h_l = 0$ and

$$\sum_{l=0}^{L-1} h_l h_{l+2n} = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases} \tag{IA.1}$$

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where the second summation expresses the orthonormality property of a wavelet filter. In the above $h_l \equiv 0$ for all $l < 0$ and $l \geq L$ so we actually consider $\{h_l\}$ to be an infinite sequence with at most L non-zero values.

The scaling filter is defined in terms of the wavelet filter via the **quadrature mirror relationship**

$$g_l \equiv (-1)^{l+1} h_{L-1-l} \qquad \text{for } l = 0, \ldots, L-1. \tag{IA.2}$$

This filter satisfies the conditions

$$\sum_{l=0}^{L-1} g_l g_{l+2n} = \begin{cases} 1, & n = 0 \\ 0, & otherwise \end{cases} \tag{IA.3}$$

and $\sum_{l=0}^{L-1} g_l h_{l+2n} = 0$ for all n. Without loss of generality, we can also assume that $\sum_l g_l = \sqrt{2}$.

In discrete time, the **frequency response function** or **transfer function** for $\{h_l\}$ is defined as

$$H(f) \equiv \sum_{l=-\infty}^{\infty} h_l e^{-i2\pi fl} = \sum_{l=0}^{L-1} h_l e^{-i2\pi fl} \tag{IA.4}$$

where $i = \sqrt{-1}$ and f is the frequency defined as the inverse of the cycle length (period) $f = 1/p$ where f is the number of cycles per unit time. The frequency $f = 1/2$ is the highest possible frequency since the shortest length of a cycle would be two time periods. If we use $\mathcal{H}(f) \equiv |H(f)|^2$ to define the associated **squared gain function** then the orthonormality property is equivalent to

$$\mathcal{H}(f) + \mathcal{H}(f + \frac{1}{2}) = 2 \qquad \text{for all } f. \tag{IA.5}$$

If we let $G(\cdot)$ and $\mathcal{G}(\cdot)$ be the transfer and squared gain functions for the scaling filter, we have

$$\mathcal{G}(f) + \mathcal{G}(f + \frac{1}{2}) = 2 \qquad \text{and} \qquad \mathcal{H}(f) + \mathcal{G}(f) = 2 \qquad \text{for all } f. \tag{IA.6}$$

In practice, $\{h_l\}$ is nominally a high-pass filter with a pass-band given by $\frac{1}{4} \leq |f| \leq \frac{1}{2}$, while $\{g_l\}$ is nominally a low-pass filter with pass band $0 \leq |f| \leq \frac{1}{4}$.

Now let $\left\{h_{j,l}\right\}$ and $\left\{g_{j,l}\right\}$ be the j − th level DWT equivalent wavelet and scaling filters each having width $L_j \equiv \left(2^j - 1\right)(L-1) + 1$. Here $h_{1,l} \equiv h_l$ and $g_{1,l} \equiv g_l$. These filters have transfer functions

$$H_j(f) \equiv H\left(2^{j-1} f\right) \prod_{l=0}^{j-2} G\left(2^l f\right) \quad \text{and} \quad G_j(f) \equiv \prod_{l=0}^{j-1} G\left(2^l f\right) \tag{IA.7}$$

where $H_1(f) \equiv H(f)$ and $G_1(f) \equiv G(f)$. The filter $\left\{h_{j,l}\right\}$ is nominally a band-pass filter with pass-band given by $1/2^{j+1} \leq |f| \leq 1/2^j$ while $\left\{g_{j,l}\right\}$ is nominally a low-pass filter with pass-band $0 \leq |f| \leq 1/2^{j+1}$.

Let $\left\{\widetilde{h}_{j,l}\right\}$ and $\left\{\widetilde{g}_{j,l}\right\}$ be the j − th level MODWT wavelet and scaling filters. These filters are defined in terms of the j − th level equivalent wavelet and scaling filters $\left\{h_{j,l}\right\}$ and $\left\{g_{j,l}\right\}$ for the DWT as $\widetilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}}$ and $\widetilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}$. Each of the MODWT filters has width $L_j \equiv \left(2^j - 1\right)(L-1)+1$ and can be generated once a basic MODWT wavelet filter $\widetilde{h}_{1,l} \equiv \widetilde{h}_l \equiv h_l/\sqrt{2}$ and its related MODWT scaling filter $\widetilde{g}_{1,l} \equiv \widetilde{g}_l \equiv (-1)^{l+1}\widetilde{h}_{L-1-l}$ have been specified. Specifically, if $\widetilde{H}(\cdot)$ and $\widetilde{G}(\cdot)$ are the transfer functions for $\left\{\widetilde{h}_l\right\}$ and $\{\widetilde{g}_l\}$, then the transfer functions for $\left\{\widetilde{h}_{j,l}\right\}$ and $\left\{\widetilde{g}_{j,l}\right\}$ are given by

$$\widetilde{H}_j(f) \equiv \widetilde{H}\left(2^{j-1} f\right) \prod_{l=0}^{j-2} \widetilde{G}\left(2^l f\right) \quad \text{and} \quad \widetilde{G}_j(f) \equiv \prod_{l=0}^{j-1} \widetilde{G}\left(2^l f\right). \tag{IA.8}$$

Table IA.1 provides a summary of key relationships involving DWT and MODWT wavelet and scaling filters.

Table IA.1: Wavelet Cheat Sheet: Summary of Key Relationships

DWT and MODWT wavelet filters	DWT and MODWT scaling filters
definitions, quadrature mirror relationship and frequency response functions	
$\{h_l\} \longleftrightarrow H(\cdot)$, $\widetilde{h}_l \equiv h_l/\sqrt{2}$ and $\left\{\widetilde{h}_l\right\} \longleftrightarrow \widetilde{H}(\cdot) = \frac{1}{\sqrt{2}}H(\cdot)$	$\{g_l\} \longleftrightarrow G(\cdot)$, $\widetilde{g}_l \equiv g_l/\sqrt{2}$ and $\{\widetilde{g}_l\} \longleftrightarrow \widetilde{G}(\cdot) = \frac{1}{\sqrt{2}}G(\cdot)$
$h_l = (-1)^l g_{L-1-l}$, $\widetilde{h}_l = (-1)^l \widetilde{g}_{L-1-l}$	$g_l = (-1)^{l+1} h_{L-1-l}$, $\widetilde{g}_l = (-1)^{l+1} \widetilde{h}_{L-1-l}$
$H(f) = \sum_{l=0}^{L-1} h_l e^{-i2\pi f} = -e^{-i2\pi f(L-1)} G\left(\frac{1}{2} - f\right)$	$G(f) = \sum_{l=0}^{L-1} g_l e^{-i2\pi f} = e^{-i2\pi f(L-1)} H\left(\frac{1}{2} - f\right)$
$\widetilde{H}(f) = -e^{-i2\pi f(L-1)} \widetilde{G}\left(\frac{1}{2} - f\right)$	$\widetilde{G}(f) = e^{-i2\pi f(L-1)} \widetilde{H}\left(\frac{1}{2} - f\right)$
$h_{1,l} \equiv h_l$, $H_1(f) \equiv H(f)$, $\widetilde{h}_{1,l} \equiv \widetilde{h}_l$, $\widetilde{H}_1(f) \equiv \widetilde{H}(f)$	$g_{1,l} \equiv g_l$, $G_1(f) \equiv G(f)$, $\widetilde{g}_{1,l} \equiv \widetilde{g}_l$, $\widetilde{G}_1(f) \equiv \widetilde{G}(f)$
$H_j(f) \equiv H\left(2^{j-1} f\right) \prod_{l=0}^{j-2} G\left(2^l f\right)$, $\widetilde{H}_j(f) \equiv \widetilde{H}\left(2^{j-1} f\right) \prod_{l=0}^{j-2} \widetilde{G}\left(2^l f\right)$	$G_j(f) \equiv \prod_{l=0}^{j-1} G\left(2^l f\right)$, $\widetilde{G}_j(f) \equiv \prod_{l=0}^{j-1} \widetilde{G}\left(2^l f\right)$
$H_j(f) \equiv H\left(2^{j-1} f\right) G_{j-1}(f)$, $\widetilde{H}_j(f) \equiv \frac{1}{2^{j/2}} H_j(f)$	$G_j(f) \equiv G\left(2^{j-1} f\right) G_{j-1}(f)$, $\widetilde{G}_j(f) \equiv \widetilde{G}\left(2^{j-1} f\right) \widetilde{G}_{j-1}(f)$
$\{h_{j,l}\} \longleftrightarrow H_j(\cdot)$, $\left\{\widetilde{h}_{j,l}\right\} \longleftrightarrow \widetilde{H}_j(\cdot)$	$\{g_{j,l}\} \longleftrightarrow G_j(\cdot)$, $\{\widetilde{g}_{j,l}\} \longleftrightarrow \widetilde{G}_j(\cdot)$
3 basic properties: sums, energy and orthogonality to even shifts	
$\sum_{l=0}^{L-1} h_l = H(0) \equiv 0$, $\sum_{l=0}^{L-1} \widetilde{h}_l = \widetilde{H}(0) \equiv 0$	$\sum_{l=0}^{L-1} g_l = G(0) \equiv \sqrt{2}$, $\sum_{l=0}^{L-1} \widetilde{g}_l = \widetilde{G}(0) \equiv 1$
$\sum_{l=0}^{L-1} h_l^2 = 1$, $\sum_{l=0}^{L-1} \widetilde{h}_l^2 = \frac{1}{2}$	$\sum_{l=0}^{L-1} g_l^2 = 1$, $\sum_{l=0}^{L-1} \widetilde{g}_l^2 = \frac{1}{2}$
$\sum_{l=0}^{L-1} h_l h_{l+2n} = 0, \quad n \neq 0$, $\sum_{l=0}^{L-1} \widetilde{h}_l \widetilde{h}_{l+2n} = 0, \quad n \neq 0$	$\sum_{l=0}^{L-1} g_l g_{l+2n} = 0, \quad n \neq 0$, $\sum_{l=0}^{L-1} \widetilde{g}_l \widetilde{g}_{l+2n} = 0, \quad n \neq 0$
$\sum_{l=0}^{L-1} g_l h_{l+2n} = 0$ and $\sum_{l=0}^{L-1} \widetilde{g}_l \widetilde{h}_{l+2n} = 0$	
$\sum_{l=0}^{L-1} h_{j,l} = H_j(0) = 0$, $\sum_{l=0}^{L-1} \widetilde{h}_{j,l} = \widetilde{H}_j(0) = 0$	$\sum_{l=0}^{L-1} g_{j,l} = G_j(0) = 2^{j/2}$, $\sum_{l=0}^{L-1} \widetilde{g}_{j,l} = \widetilde{G}_j(0) = 1$
$\sum_{l=0}^{L-1} h_{j,l}^2 = 1$, $\sum_{l=0}^{L-1} \widetilde{h}_{j,l}^2 = \frac{1}{2^j}$	$\sum_{l=0}^{L-1} g_{j,l}^2 = 1$, $\sum_{l=0}^{L-1} \widetilde{g}_{j,l}^2 = \frac{1}{2^j}$
$\sum_{l=0}^{L-1} h_{j,l} h_{j,l+2^j n} = 0, \quad n \neq 0$, $\sum_{l=0}^{L-1} \widetilde{h}_{j,l} \widetilde{h}_{j,l+2^j n} = 0, \quad n \neq 0$	$\sum_{l=0}^{L-1} g_{j,l} g_{j,l+2^j n} = 0, \quad n \neq 0$, $\sum_{l=0}^{L-1} \widetilde{g}_{j,l} \widetilde{g}_{j,l+2^j n} = 0, \quad n \neq 0$
$\sum_{l=0}^{L-1} g_{j,l} h_{j,l+2^j n} = 0$ and $\sum_{l=0}^{L-1} \widetilde{g}_{j,l} \widetilde{h}_{j,l+2^j n} = 0$	
squared gain functions	
$\mathcal{H}(f) \equiv \|H(f)\|^2$, $\widetilde{\mathcal{H}}(f) \equiv \|\widetilde{H}(f)\|^2 = \frac{1}{2}\mathcal{H}(f)$	$\mathcal{G}(f) \equiv \|G(f)\|^2$, $\widetilde{\mathcal{G}}(f) \equiv \|\widetilde{G}(f)\|^2 = \frac{1}{2}\mathcal{G}(f)$
$\mathcal{H}(f) + \mathcal{H}(f + \frac{1}{2}) = 2$, $\widetilde{\mathcal{H}}(f) + \widetilde{\mathcal{H}}(f + \frac{1}{2}) = 1$	$\mathcal{G}(f) + \mathcal{G}(f + \frac{1}{2}) = 2$, $\widetilde{\mathcal{G}}(f) + \widetilde{\mathcal{G}}(f + \frac{1}{2}) = 1$
$\mathcal{H}(f) + \mathcal{G}(f) = 2$ and $\widetilde{\mathcal{H}}(f) + \widetilde{\mathcal{G}}(f) = 1$	
$\mathcal{H}_j(f) \equiv \|H_j(f)\|^2$, $\widetilde{\mathcal{H}}_j(f) \equiv \|\widetilde{H}_j(f)\|^2 = \frac{1}{2^j}\mathcal{H}_j(f)$	$\mathcal{G}_j(f) \equiv \|G_j(f)\|^2$, $\widetilde{\mathcal{G}}_j(f) \equiv \|\widetilde{G}_j(f)\|^2 = \frac{1}{2^j}\mathcal{G}_j(f)$
pyramid algorithm and direct definition	
$W_{j,t} \equiv \sum_{l=0}^{L-1} h_l V_{j-1,2t+1-l \bmod N_{j-1}}$, $\widetilde{W}_{j,t} \equiv \sum_{l=0}^{L-1} \widetilde{h}_l \widetilde{V}_{j-1,t-2^{j-1}l \bmod N}$	$V_{j,t} \equiv \sum_{l=0}^{L-1} g_l V_{j-1,2t+1-l \bmod N_{j-1}}$, $\widetilde{V}_{j,t} \equiv \sum_{l=0}^{L-1} \widetilde{g}_l \widetilde{V}_{j-1,t-2^{j-1}l \bmod N}$
$W_{1,t} \equiv \sum_{l=0}^{L-1} h_l X_{2t+1-l \bmod N}$, $\widetilde{W}_{1,t} \equiv \sum_{l=0}^{L-1} \widetilde{h}_l X_{t-l \bmod N}$	$V_{1,t} \equiv \sum_{l=0}^{L-1} g_l X_{2t+1-l \bmod N}$, $\widetilde{V}_{1,t} \equiv \sum_{l=0}^{L-1} \widetilde{g}_l X_{t-l \bmod N}$
$W_{j,t} \equiv \sum_{l=0}^{L_j-1} h_{j,l} X_{2^j(t+1)-1-l \bmod N}$, $\widetilde{W}_{j,t} \equiv \sum_{l=0}^{L_j-1} \widetilde{h}_{j,l} X_{t-l \bmod N}$	$V_{j,t} \equiv \sum_{l=0}^{L_j-1} g_{j,l} X_{2^j(t+1)-1-l \bmod N}$, $\widetilde{V}_{j,t} \equiv \sum_{l=0}^{L_j-1} \widetilde{g}_{j,l} X_{t-l \bmod N}$

Notes: Key relationships involving DWT and MODWT wavelet and scaling filters. Because $h_l = g_l = 0$ for all $l < 0$ and $l \geq L$, summations involving h_l or g_l can be taken to range from either $l = 0$ to $l = L - 1$ or over all integers; summations involving $h_{j,l}$ or $g_{j,l}$ can range either from $l = 0$ to $l = L_j - 1$ or over all integers. Note that $L_j \equiv \left(2^j - 1\right)(L - 1) + 1$ and $N_j \equiv N/2^j$.

1.2 Discrete Wavelet Families

Our discussion closely follow Percival and Walden (2000). For a more mathematically rigorous treatment see also Daubechies (1992, Chapter 6) and Mallat (2008, Chapter 7). A **Daubechies wavelet** filter of even width L has a squared gain function given by

$$\mathcal{H}^{\mathcal{D}}(f) \equiv \mathcal{D}^{\frac{L}{2}}(f)\,\mathcal{A}_L(f) \tag{IA.9}$$

where $\mathcal{D}(f) \equiv 4\sin^2(\pi f)$ defines the squared gain function for the difference filter $\{1, -1\}$ and

$$\mathcal{A}_L(f) \equiv \frac{1}{2^{L-1}} \sum_{l=0}^{L/2-1} \binom{L/2-1+l}{l} \cos^{2l}(\pi f) \tag{IA.10}$$

which constitutes the squared gain function of a low-pass filter. Equation (IA.9) indicates that the Daubechies wavelet filter filters can be interpreted as the equivalent filter for a filter cascade consisting of $\frac{L}{2}$ difference filters (yielding the overall differencing operation) along with a low-pass filter (yielding the weighted average). The scaling filter $\{g_l\}$ that corresponds to a Daubechies wavelet filter has a squared gain function given by

$$\mathcal{G}^{\mathcal{D}}(f) \equiv \mathcal{H}^{\mathcal{D}}\left(\frac{1}{2} - f\right) = 2\cos^L(\pi f) \sum_{l=0}^{L/2-1} \binom{L/2-1+l}{l} \sin^{2l}(\pi f) \tag{IA.11}$$

The Daubechies wavelet filter of width $L = 2$ is the Haar wavelet filter $\left\{h_0 = \frac{1}{\sqrt{2}}, h_1 = -\frac{1}{\sqrt{2}}\right\}$.

In general, there are several real-valued wavelet filters of the form $\{h_l : l = 0, \ldots, L-1\}$ with the same squared gain function $\mathcal{H}^{\mathcal{D}}(\cdot)$. We need to impose additional criterion to select a unique wavelet or scaling filter as L increases.

The first criterion is to pick the scaling filter $\left\{g_l^{ep}\right\}$ with squared gain $\mathcal{G}^{\mathcal{D}}(\cdot)$ such that

$$\sum_{l=0}^{m} g_l^2 \leq \sum_{l=0}^{m} \left[g_l^{ep}\right]^2 \quad for \quad m = 0, \ldots, L-1 \tag{IA.12}$$

where $\{g_l\}$ is any other filter with squared gain $\mathcal{G}^{\mathcal{D}}(\cdot)$. This extremal phase choice yields a scaling filter with minimum delay. We denote the filters satisfying this criterion as the $D(L)$ filters $L = 2, 4, \ldots$. Filters of this form have their energy concentrated near the starting point of their support. As a result they are highly nonsymmetric which yields asymmetric wavelets.

The second criterion is to pick the scaling filter whose transfer function

$$G(f) = \left[\mathcal{G}^{\mathcal{D}}(f)\right]^{1/2} e^{i\theta^{\mathcal{G}}(f)} \tag{IA.13}$$

has a phase function $\theta^{\mathcal{G}}(\cdot)$ that is as close as possible to that of a linear phase filter. In particular, for a given shift $\bar{v}$ we compute

$$\rho_{\bar{v}}(\{g_l\}) \equiv \max_{-\frac{1}{2} \leq f \leq \frac{1}{2}} |\theta^{\mathcal{G}}(f) - 2\pi f \bar{v}| \tag{IA.14}$$

where $\theta^{\mathcal{G}}(\cdot)$ is the phase function for the $\{g_l\}$ under consideration. Let v be the shift that minimizes the above - hence $\theta^{\mathcal{G}}(f) \simeq 2\pi f v$. The **least asymmetric filter** (also referred to as **symmlet**) is the one such that $\rho_v(\{g_l\})$ is as small as possible. We denote the filters satisfying this criterion as the $LA(L)$ filters $L = 8, 10, \ldots$ where LA stands for least asymmetric. The advantage of the LA filters is that we can use the value that minimizes the above so align both the scaling and wavelet coefficients such that they can be regarded as approximately the output zero phase filters. This approximate zero phase property is important because it allows us to meaningfully relate DWT coefficients to various events in the original time series.

The **Coiflet wavelet** filters denoted as $C(L)$ with $L = 6, 12, 18, 24, 30$ are alternatives to the

Daubechies filters that provide better approximations to zero phase filters than the LA filters do. The squared gain function for $\{h_l^C\}$ can be written as

$$\mathcal{H}^C(f) = \mathcal{D}^{\frac{L}{3}}(f)\left(\sum_{l=0}^{L/6-1}\begin{pmatrix} L/6-1+l \\ l \end{pmatrix}\cos^{2l}(\pi f) + \cos^{\frac{L}{3}}(\pi f)\, F(f)\right)^2 \tag{IA.15}$$

where $F(\cdot)$ is a trigonometric polynomial chosen to force the condition

$$\mathcal{H}^C(f) + \mathcal{H}^C(f + \frac{1}{2}) = 2 \qquad \text{for all } f. \tag{IA.16}$$

Whereas the Daubechies wavelet filters of width L have $L/2$ embedded differencing operations, the coiflet filters involve $L/3$ such differences so there are more terms devoted to the averaging portion of the filter. The phase function for $\{g_l^C\}$ is approximately given by $2\pi f v$ with $v = -\frac{2L}{3} + 1$.

Table IA.2 provides a summary of properties of the different wavelet families.

Table IA.2: Properties of Discrete Wavelet Families

Property / Filter	Haar	Daubechies	Least Asymmetric	Coiflet
Symmetry	✓			
Asymmetry		✓		
Near symmetry			✓	✓
Orthogonal analysis	✓	✓	✓	✓
Exact reconstruction	✓	✓	✓	✓
Explicit expression	✓			

Notes: Summary of properties for discrete wavelet families.

2 Further Discussion: Wavelet-Based Beta Decompositions with Details and Smooths

In this section we proceed to show that: (i) $\| \widetilde{\boldsymbol{D}}_j \|^2 \leq \| \widetilde{\boldsymbol{W}}_j \|^2$, $\| \widetilde{\boldsymbol{S}}_j \|^2 \leq \| \widetilde{\boldsymbol{V}}_j \|^2$ and (ii) $\| \widetilde{\boldsymbol{D}}_{j,n} \|^2 \leq \| \widetilde{\boldsymbol{W}}_{j,n} \|^2$. These results explain why we are not defining beta decompositions between the returns of an asset and a candidate factor using the corresponding wavelets details (or smooths) of the two in the direct approach, i.e. covariances of the form $\mathrm{Cov}\left(\widetilde{D}^{f}_{j,t}, \widetilde{D}^{R^e_i}_{j,t}\right)$. Note that the first result is presented in Percival and Walden (2000) but the proof is left implied and might not be trivial for some readers. To our knowledge, the second result is not discussed in the wavelet literature. For completeness, we provide a thorough discussion of both.

We begin with a short reminder of basic Fourier Theory. Let $\{a_t\} = \{a_t : t = 0, \ldots, N-1\}$ be a sequence of N real-valued variables. Its discrete Fourier Transform (DFT) is the sequence $\{A_k\}$ of N variables given by

$$A_k \equiv \sum_{t=0}^{N-1} a_t e^{-i2\pi tk/N} \quad k = 0, \ldots, N-1 \tag{IA.17}$$

while the inverse DFT of $\{A_k\}$ is

$$a_t \equiv \frac{1}{N} \sum_{k=0}^{N-1} A_k e^{i2\pi tk/N} \quad t = 0, \ldots, N-1. \tag{IA.18}$$

The Fourier relationship between $\{a_t\}$ and $\{A_k\}$ is noted as $\{a_t\} \longleftrightarrow \{A_k\}$. From Parseval's theorem for finite sequences it holds that $\sum_{t=0}^{N-1} |a_t|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |A_k|^2$.

If $\{a_t : t = \ldots, -1, 0, 1, \ldots\} \longleftrightarrow A(\cdot)$, then $c_t \equiv \sum_{u=-\infty}^{\infty} a_u b_{t-u \bmod N}$, $t = 0, \ldots, N-1$ can be expressed as a circular convolution $c_t \equiv \sum_{u=0}^{N-1} a^{\circ}_u b_{t-u \bmod N}$ where $a^{\circ}_t \equiv \sum_{u=-\infty}^{\infty} a_{t+uN}$. The sequence $\left\{a^{\circ}_t\right\}$ of length N is said to be formed by periodizing $\{a_t\}$ to length N. We have $\left\{a^{\circ}_t : t = 0, \ldots, N-1\right\} \longleftrightarrow \left\{A\left(\frac{k}{N}\right) : k = 0, \ldots, N-1\right\}$.

Let $\left\{\widetilde{h}^{\circ}_{j,l}\right\}$ and $\left\{\widetilde{g}^{\circ}_{j,l}\right\}$ be the filters obtained by periodizing $\left\{\widetilde{h}_{j,l}\right\}$ and $\left\{\widetilde{g}_{j,l}\right\}$ to length N. The

DFTs of $\left\{\widetilde{h}^{\circ}_{j,l}\right\}$ and $\left\{\widetilde{g}^{\circ}_{j,l}\right\}$ are given respectively by $\widetilde{H}_j\left(\frac{k}{N}\right)$ and $\widetilde{G}_j\left(\frac{k}{N}\right)$. We can re-express

$$\widetilde{W}_{j,t} \equiv \sum_{l=0}^{N-1} \widetilde{h}^{\circ}_{j,l} X_{t-l \bmod N} \quad \text{and} \quad \widetilde{V}_{j,t} \equiv \sum_{l=0}^{N-1} \widetilde{g}^{\circ}_{j,l} X_{t-l \bmod N} \tag{IA.19}$$

$t = 0, 1, \ldots, N-1$. Letting $\{\mathcal{X}_k\}$ be the DFT of $\{X_t\}$ we have $\left\{\widetilde{W}_{j,t}\right\} \longleftrightarrow \left\{\widetilde{H}_j\left(\frac{k}{N}\right)\mathcal{X}_k\right\}$ and $\left\{\widetilde{V}_{j,t}\right\} \longleftrightarrow \left\{\widetilde{G}_j\left(\frac{k}{N}\right)\mathcal{X}_k\right\}$. Parseval's theorem gives us that

$$\| \widetilde{\boldsymbol{W}}_j \|^2 = \frac{1}{N}\sum_{k=0}^{N-1} \underbrace{|\widetilde{H}_j\left(\frac{k}{N}\right)|^2}_{=\widetilde{\mathcal{H}}_j\left(\frac{k}{N}\right)} |\mathcal{X}_k|^2 \quad \text{and} \quad \| \widetilde{\boldsymbol{V}}_j \|^2 = \frac{1}{N}\sum_{k=0}^{N-1} \underbrace{|\widetilde{G}_j\left(\frac{k}{N}\right)|^2}_{=\widetilde{\mathcal{G}}_j\left(\frac{k}{N}\right)} |\mathcal{X}_k|^2. \tag{IA.20}$$

We can also re-express $\widetilde{\boldsymbol{D}}_j$ and $\widetilde{\boldsymbol{S}}_j$ as

$$\widetilde{D}_{j,t} \equiv \sum_{l=0}^{N-1} \widetilde{h}^{\circ}_{j,l} \widetilde{W}_{j,t+l \bmod N} \quad \text{and} \quad \widetilde{S}_{j,t} \equiv \sum_{l=0}^{N-1} \widetilde{g}^{\circ}_{j,l} \widetilde{V}_{j,t+l \bmod N} \tag{IA.21}$$

$t = 0, 1, \ldots, N-1$. Letting $\{\mathcal{X}_k\}$ represent the DFT of $\{X_t\}$ it follows that

$$\left\{\widetilde{D}_{j,t}\right\} \longleftrightarrow \left\{\widetilde{H}^*_j\left(\frac{k}{N}\right)\widetilde{H}_j\left(\frac{k}{N}\right)\mathcal{X}_k\right\} = \left\{|\widetilde{H}_j\left(\frac{k}{N}\right)|^2\mathcal{X}_k\right\} \tag{IA.22}$$

where $\widetilde{H}^*(\cdot)$ is the complex conjugate of $\widetilde{H}(\cdot)$ and similarly $\left\{\widetilde{S}_{j,t}\right\} \longleftrightarrow \left\{|\widetilde{G}_j\left(\frac{k}{N}\right)|^2\mathcal{X}_k\right\}$. The energy for the $j-$th level detail and smooth is given by

$$\| \widetilde{\boldsymbol{D}}_j \|^2 = \frac{1}{N}\sum_{k=0}^{N-1} \widetilde{\mathcal{H}}^2_j\left(\frac{k}{N}\right)|\mathcal{X}_k|^2 \quad \text{and} \quad \| \widetilde{\boldsymbol{S}}_j \|^2 = \frac{1}{N}\sum_{k=0}^{N-1} \widetilde{\mathcal{G}}^2_j\left(\frac{k}{N}\right)|\mathcal{X}_k|^2. \tag{IA.23}$$

Since $\mathcal{H}_j(f) + \mathcal{G}_j(f) = 1$ it follows that $\mathcal{H}_j(f) \leq 1$ and $\mathcal{G}_j(f) \leq 1$ and in general $\| \widetilde{\boldsymbol{D}}_j \|^2 \leq \| \widetilde{\boldsymbol{W}}_j \|^2$ and $\| \widetilde{\boldsymbol{S}}_j \|^2 \leq \| \widetilde{\boldsymbol{V}}_j \|^2$. Note that $\| \widetilde{\boldsymbol{D}}_j \|^2 = \| \widetilde{\boldsymbol{W}}_j \|^2$ and $\| \widetilde{\boldsymbol{S}}_j \|^2 = \| \widetilde{\boldsymbol{V}}_j \|^2$ only asymptotically as $L \longrightarrow \infty$ (see Lai, 1995).

From Equation (5) $\widetilde{\boldsymbol{W}}_{j,n}$ is the result of circularly convolving $\boldsymbol{X}$ with the filter $\widetilde{u}_{j,n,l}$, implicitly periodized to length N. Parseval's theorem gives us that

$$\| \widetilde{\boldsymbol{W}}_{j,n} \|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |\widetilde{U}_{j,n}\left(\frac{k}{N}\right)|^2 |\mathcal{X}_k|^2. \tag{IA.24}$$

The DFT of $\widetilde{\boldsymbol{D}}_{j,n}$ is given by $\left\{\widetilde{\boldsymbol{D}}_{j,n}\right\} \longleftrightarrow \left\{|\widetilde{U}_{j,n}\left(\frac{k}{N}\right)|^2 \mathcal{X}_k\right\}$. The energy of the details $\widetilde{\boldsymbol{D}}_{j,n}$ in each frequency band is given by

$$\| \widetilde{\boldsymbol{D}}_{j,n} \|^2 = \frac{1}{N} \sum_{k=0}^{N-1} \left(|\widetilde{U}_{j,n}\left(\frac{k}{N}\right)|^2\right)^2 |\mathcal{X}_k|^2. \tag{IA.25}$$

Since $\sum_{n=0}^{2^j-1} |\widetilde{U}_{j,n,k}|^2 = 1$ (see Walden and Contreras-Cristan, 1998, pages 2250-51), it follows that $\left(|\widetilde{U}_{j,n}\left(\frac{k}{N}\right)|^2\right)^2 \leq |\widetilde{U}_{j,n}\left(\frac{k}{N}\right)|^2$ and $\| \widetilde{\boldsymbol{D}}_{j,n} \|^2 \leq \| \widetilde{\boldsymbol{W}}_{j,n} \|^2$.

3 Frequency-specific DGP with a Useful Factor

The central idea in the line of work that explores heterogeneity in the pricing of risk over frequencies (see Bandi and Tamoni 2013; Boons and Tamoni 2015; Xyngis 2017; Kang et al. 2017; Bandi et al. 2021) is that **assets offer different risk premia because they are differentially exposed to a (low-frequency) factor that is priced at a specific frequency-band**. To capture this idea we use the following data generating process based on the MODWPT: We simulate a single realization for a zero-mean normally distributed useful factor $f \sim N(0, \sigma^2_{MKT})$ with variance calibrated to the market factor. The mean of the useful factor is not important because all wavelet packet coefficients are zero-mean by construction. For a specific j and for $n^\star \in \left\{0, \ldots, 2^j - 1\right\}$ we let

$$\widetilde{W}^{R_i^s}_{j,n^\star,t} = \beta^\top_{i,\widetilde{W}_{j,n^\star}} \widetilde{W}^f_{j,n^\star,t} \tag{IA.26}$$

while for $n \in \{0, \ldots, 2^j - 1\}$ with $n \neq n^\star$ we assume that $\widetilde{W}^{R_i^s}_{j,n,t} \sim N(0, \sigma_n^2)$ where σ_n^2 denotes the frequency-specific variance of the process at band n. Then, we inverse the decomposition up to scale $j = 0$ and recover the raw time-series of returns for each portfolio (we apply the inverse decomposition as in Walden and Contreras-Cristan, 1998, page 2251). We alter the average return of each portfolio by adding $\gamma_1^\intercal \beta_{i,\widetilde{W}_{j,n^\star}}$, i.e. $\mu_{R_i^s} = \gamma_1^\intercal \beta_{i,\widetilde{W}_{j,n^\star}}$. As a result, these assets have different average returns only because they have different betas with respect to the useful factor.

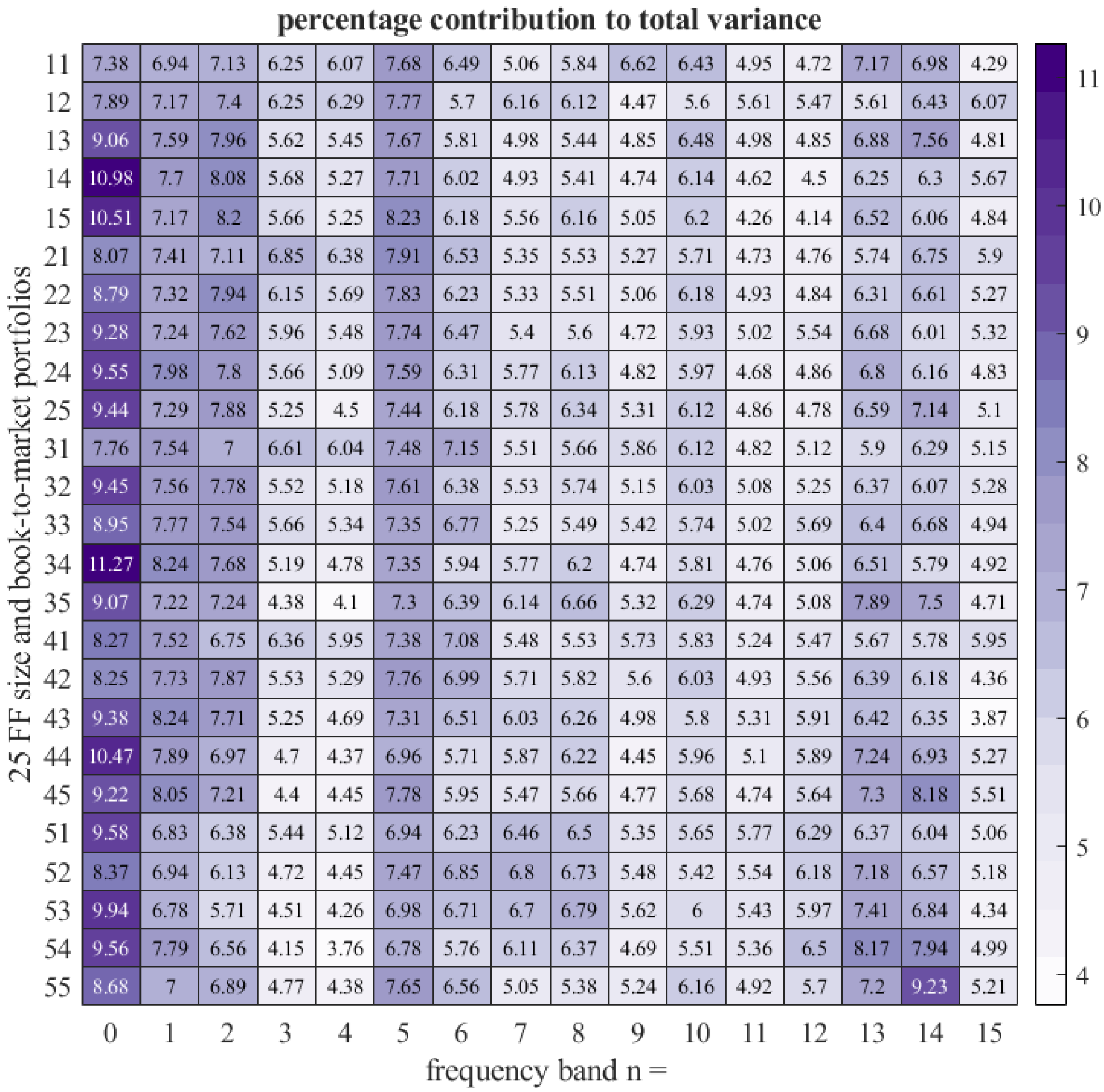
Calibration details: We set $j = 4$ and split the frequency band into 16 equal intervals with $n = 0, \ldots, 15$. We calibrate σ_n^2 using frequency-specific estimates of the variances of the 25 Fama–French size and book-to-market portfolios based on the MODWPT in line with Walden and Contreras-Cristan (1998). Figure IA.1 presents the frequency-specific percentage contribution of each wavelet packet component to the total variance of the series for all of the 25 size and book-to-market portfolios. We calibrate the frequency-specific betas $\beta_{i,W_{j,n^\star}}$ using betas estimates from a simple time-series regression between the market factor and the 25 FF size and book-to-market portfolios. We set $\gamma_1 = 2$ and simulate 5,000 sets of returns for 25 portfolios (i.e., $R_i^s, i = 1, \ldots, 25$) with $N = 1024, 256$ observations. We set $n^\star = 15$ to localize the useful factor at high frequencies, $n^\star = 0$ to localize the useful factor at low frequencies and $n^\star = 7$ for medium frequencies.

4 Misspecification Robust Bootstrap and Rank Tests

4.1 Bootstrapping

We stack the k factors and N_A test assets in a $N \times (N_A + k)$ matrix Z with rows $z_t = [f_t^\intercal, r_t^\intercal]$ for $t = 1, \ldots, N$. The bootstrap samples $\{z_t^*\}_{t=1}^N$ are constructed by drawing with replacement blocks of m $(1 \leq m \leq N)$ observations from matrix Z. We use the circular block bootstrap. The block length m is estimated based on the Politis and White (2004) estimator for the optimal block size.

Figure IA.1: Simulation Details: Frequency-specific Contribution to Total Variance



Notes: This figure (heatmap) presents the frequency-specific percentage contribution of each wavelet packet component to the total variance of the series for the 25 Fama–French size and book-to-market portfolios based on the MODWPT in line with Walden and Contreras-Cristan (1998).

We round the estimate UP to the nearest integer in line with the recommendation of Politis and White (2004). The bootstrap p-value for the test of statistical significance is constructed by bootstrapping the standardized (asymptotically pivotal) statistic. In particular, let $\hat{t}_i = \sqrt{N}\hat{\gamma}_i / \sqrt{\hat{V}_{ii}(\hat{\gamma})}$ denote the sample MR t-test for parameter γ_i where $\hat{V}_{ii}(\hat{\gamma})$ is the (i, i) -th element of the estimated misspecification-robust covariance matrix and let

$$\hat{t}^*_{i,b} = \sqrt{N}\left(\hat{\gamma}^*_i - \hat{\gamma}_i\right)) / \sqrt{\hat{V}^*_{ii}(\hat{\gamma}^*)} \tag{IA.27}$$

be its bootstrap analogue for the b-th bootstrap sample. The bootstrap p-value is given by $\frac{1}{B}\sum_{b=1}^{B} \mathbb{1}\left\{\hat{t}^*_{i,b} > \hat{t}_i\right\}$. With the exception of the block size, our approach follows Gospodinov and Robotti (2021).

4.2 Rank Tests

To test whether the $N_A \times (k+1)$ matrix $X = \left(1_{N_A}, \beta\right)$ has a full column rank we apply a family of rank tests introduced by Gospodinov et al. (2017) and Gospodinov and Robotti (2021). In particular, Gospodinov et al. (2017) note that a convenient way of testing for rank deficiency of X is to remove the column of 1s by multiplying by P where P is an $N_A \times (N_A - 1)$ orthonormal matrix whose column are orthogonal to 1_{N_A}. That is, they suggest performing a rank test on the $(N_A - 1) \times k$ matrix $\Pi = P^\top \beta$. Let $\hat{\Pi} = P^\top \hat{\beta}$, then

$$\sqrt{N} vec\left(\hat{\Pi} - \Pi\right) \xrightarrow{d} N\left(0_{(N_A-1)k}, S_{\hat{\Pi}}\right) \tag{IA.28}$$

where $S_{\hat{\Pi}} = \sum_{j=-\infty}^{+\infty} E\left[\tilde{x}_t \tilde{x}_{t+j}^\top\right]$ and $\tilde{x}_t = V_f^{-1}\left(f_t - \mu_f\right) \otimes P^\top \epsilon_t$. Gospodinov et al. (2017) suggest testing H_0 : rank $(\Pi) = k - 1$ using

$$\mathcal{J}_1 = N \min_c \left[-1, c^\top\right] \hat{\Pi}^\top \left(C \hat{S}_{\hat{\Pi}} C^\top\right)^{-1} \hat{\Pi} \left[-1, c^\top\right]^\top \overset{A}{\sim} \chi^2_{N_A - k} \tag{IA.29}$$

where c is a $k-1$ vector and $C = \left[-1, c^\top\right] \otimes I_{N_A - 1}$. In addition, Gospodinov and Robotti (2021) show that if $\tilde{x}_t$ is serially uncorrelated and $\text{Var}\left[r_t | f_t\right] = \Sigma = V_r - \beta V_f \beta^\top$, then $S_{\hat{\Pi}} = V_f^{-1} \otimes P^\top \Sigma P$. In line with Gospodinov and Robotti (2021), the (approximate) F-test of H_0 : rank $(X) = k$ that we use in the paper is given by

$$\mathcal{J}_2 = \xi_k \left(N - N_A\right) / \left(N_A - k\right) \sim F_{N_A - k, N - N_A} \tag{IA.30}$$

where ξ_k is the smallest eigenvalue of $\hat{V}_f \hat{\beta}^\top P \left(P^\top \hat{\Sigma} P\right)^{-1} P^\top \hat{\beta}$ and $\hat{\Sigma} = \hat{V}_r - \hat{\beta} \hat{V}_f \hat{\beta}^\top$. When $k = 1$ the p-value for $\mathcal{J}_2$ is exact.

5 Additional Results

5.1 Additional Simulation Results

Tables IA.6a to IA.6c report rejection rates for simulated returns with a frequency-specific DGP and N = 256.

5.2 Different Test Assets

We explore the robustness of our results with respect to two alternative sets of test portfolios, namely the 25 Fama and French (2015) size and investment & size and operating profitability portfolios.

- Tables IA.8 and IA.9 present results for the Jurado et al. (2015) macro uncertainty index.
- Tables IA.10 and IA.11 present results for industrial production growth.
- Tables IA.12 and IA.13 present results for the volatility of industrial production growth.
- Tables IA.14 to IA.16 present results for the Jurado et al. (2015) financial uncertainty index across all test assets.

- Tables IA.17 to IA.19 present results for the Jurado et al. (2015) real uncertainty index across all test assets.

- Tables IA.20 to IA.22 present results for consumption growth across all test assets.

5.3 MODWT MRA with Boundary Independent Coefficients Only

Tables IA.23a to IA.23c report additional results for a beta decomposition based on the MODWT MRA **indirect** method using boundary independent coefficients only.

5.4 Tests for Rank Failure

The condition that $X = (1_{N_A}, \beta)$ is of full column rank is crucial for valid statistical inference under Fama and MacBeth (1973) and standard bootstrapping methods that (explicitly or implicitly) maintain the identification assumption. Table IA.26 reports rejection rates at a 5% significance level for the null hypothesis that the $N_A \times (k+1)$ matrix X is of a reduced rank, i.e. $H_0 : \text{rank}(X) = k$. We report results for two rank tests: $\mathcal{J}_1$ from Gospodinov et al. (2017) that allows for conditional heteroskedasticity (see Equation IA.29) and $\mathcal{J}_2$ from Gospodinov and Robotti (2021) (see Equation IA.30). For a useless factor, the rejection rates for the second test are very close to their nominal size. In contrast, $\mathcal{J}_1$ significantly overrejects the null. For a useful factor, we find that both tests significantly overreject the null of a reduced rank at adjacent frequency bands when the factor is priced at low and medium level frequencies (note that this is in line with the simulation results in the main paper). We conclude that for empirical applications with spectral betas, $\mathcal{J}_2$ is a useful pre-test for possible identification problems.

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Table IA.3: Rejection Rates with Simulated Returns in the Spectral Factor Model of Bandi et al. (2021)

Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	>5 $\left[\frac{1}{64},0\right]$	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	>5 $\left[\frac{1}{64},0\right]$
		N = 1024						**N = 256**					
Panel A		Priced factor is at a frequency band $f \in [0.469, 0.50]$											
$H_0:\lambda_1=0$	0.10	1.0000	0.1680	0.1280	0.0420	0.0090	0.0490	1.0000	0.0870	0.0550	0.0960	0.0210	0.0610
	0.05	1.0000	0.1050	0.0720	0.0180	0.0030	0.0240	1.0000	0.0520	0.0350	0.0610	0.0120	0.0300
	0.01	1.0000	0.0370	0.0190	0.0060	0.0000	0.0030	0.9980	0.0250	0.0060	0.0210	0.0040	0.0020
Average CSR R^2		95.03%	5.68%	5.07%	4.72%	4.68%	4.29%	91.01%	4.94%	4.41%	4.62%	4.65%	4.58%
Median CSR R^2		95.26%	3.11%	2.48%	2.40%	2.27%	2.25%	91.26%	2.34%	2.26%	2.43%	2.18%	2.17%
Panel B		Priced factor is at a frequency band $f \in [0, 0.031]$											
$H_0:\lambda_1=0$	0.10	0.1910	0.1800	0.0700	0.3280	1.0000	1.0000	0.2070	0.1600	0.0710	0.0010	0.5140	1.0000
	0.05	0.1250	0.1200	0.0430	0.2520	0.9850	1.0000	0.1400	0.1030	0.0400	0.0000	0.4070	1.0000
	0.01	0.0400	0.0460	0.0150	0.1240	0.5270	1.0000	0.0680	0.0460	0.0170	0.0000	0.2620	1.0000
Average CSR R^2		6.89%	7.31%	5.11%	31.44%	84.26%	99.24%	4.79%	4.64%	5.59%	3.87%	38.60%	98.62%
Median CSR R^2		4.19%	4.09%	2.81%	31.12%	84.57%	99.26%	2.21%	2.20%	2.95%	1.85%	38.87%	98.65%
Panel C		Priced factor is at a frequency band $f \in [0.219, 0.250]$											
$H_0:\lambda_1=0$	0.10	0.9330	1.0000	0.1880	0.0720	0.0240	0.0620	0.5370	1.0000	0.0580	0.0050	0.0210	0.0860
	0.05	0.9000	1.0000	0.1220	0.0260	0.0080	0.0270	0.4650	1.0000	0.0320	0.0020	0.0070	0.0440
	0.01	0.7810	1.0000	0.0490	0.0020	0.0000	0.0030	0.3710	1.0000	0.0110	0.0010	0.0010	0.0140
Average CSR R^2		34.81%	90.59%	7.19%	7.17%	5.87%	4.32%	25.76%	90.25%	4.73%	5.23%	4.90%	4.68%
Median CSR R^2		35.01%	90.81%	3.93%	4.18%	3.32%	2.21%	24.68%	90.71%	2.32%	2.90%	2.50%	2.21%

Notes: This table reports the probability of rejecting $H_0:\lambda_1=0$ (price of covariance risk) using a two-tailed t-test at various significance levels for a useful factor in the Spectral Factor Model of Bandi et al. (2021). Inference is based on Kan et al. (2013) misspecification robust t-statistics using a heteroskedasticity and autocorrelation consistent (HAC) variance-covariance matrix with automatic lag selection and a Bartlett kernel (see Newey and West, 1994). The table also reports the average/median cross-sectional R^2. The test assets are 1,000 sets of simulated returns for 25 portfolios with $N = 1024, 256$ observations using a frequency-specific DGP based on the MODWPT. In Panel A the priced factor is at a frequency band $f \in [0.469, 0.50]$ (i.e., **high frequencies**), in Panel B the priced factor is at a frequency band $f \in [0, 0.031]$ (i.e., **low frequencies**) while in Panel C at a frequency band $f \in [0.219, 0.250]$ (i.e., **medium frequencies**).

Table IA.4: Rejection Rates with Simulated Returns and a Useless Factor: MODWT MRA

Frequency in cycles per period		$j=1$ $\frac{1}{2},\frac{1}{4}$	2 $\frac{1}{4},\frac{1}{8}$	3 $\frac{1}{8},\frac{1}{16}$	4 $\frac{1}{16},\frac{1}{32}$	5 $\frac{1}{32},\frac{1}{64}$	>5 $\frac{1}{64},0$	6 $\frac{1}{64},\frac{1}{128}$	>6 $\frac{1}{128},0$	$j=1$ $\frac{1}{2},\frac{1}{4}$	2 $\frac{1}{4},\frac{1}{8}$	3 $\frac{1}{8},\frac{1}{16}$	4 $\frac{1}{16},\frac{1}{32}$	5 $\frac{1}{32},\frac{1}{64}$	>5 $\frac{1}{64},0$	6 $\frac{1}{64},\frac{1}{128}$	>6 $\frac{1}{128},0$
		Beta decomposition based on MODWT MRA, Indirect															
		N = 1024															
Panel A		OLS								GLS							
$H_0 : \gamma_{1,j} = 0$	0.10	0.516	0.503	0.487	0.467	0.412	0.405	0.513	0.518	0.438	0.444	0.430	0.392	0.354	0.354	0.458	0.443
FM	0.05	0.436	0.425	0.400	0.373	0.325	0.324	0.432	0.443	0.354	0.360	0.347	0.312	0.268	0.270	0.375	0.365
	0.01	0.296	0.287	0.266	0.236	0.188	0.186	0.284	0.306	0.219	0.221	0.205	0.186	0.152	0.142	0.241	0.231
$H_0 : \gamma_{1,j} = 0$	0.10	0.133	0.130	0.126	0.115	0.120	0.120	0.128	0.138	0.079	0.083	0.076	0.071	0.074	0.060	0.076	0.070
MR	0.05	0.077	0.076	0.072	0.070	0.064	0.064	0.075	0.078	0.040	0.039	0.035	0.033	0.034	0.025	0.035	0.032
	0.01	0.022	0.017	0.020	0.017	0.015	0.015	0.022	0.023	0.009	0.007	0.004	0.006	0.007	0.004	0.005	0.005
$H_0 : \lambda_{1,j} = 0$	0.10	0.133	0.128	0.120	0.112	0.112	0.103	0.118	0.120	0.076	0.084	0.078	0.072	0.073	0.044	0.067	0.060
MR	0.05	0.076	0.077	0.069	0.066	0.058	0.048	0.066	0.060	0.039	0.042	0.036	0.035	0.033	0.015	0.028	0.022
	0.01	0.020	0.017	0.018	0.015	0.010	0.007	0.013	0.012	0.008	0.008	0.006	0.005	0.005	0.002	0.004	0.003
Average CSR R^2		0.136	0.135	0.134	0.131	0.138	0.135	0.133	0.136	0.041	0.042	0.041	0.040	0.042	0.041	0.043	0.041
Median CSR R^2		0.082	0.081	0.079	0.078	0.085	0.082	0.080	0.083	0.020	0.021	0.020	0.019	0.020	0.019	0.021	0.019
H_0 : CSR $R^2 = 1$		0.516	0.518	0.489	0.436	0.265	0.267	0.516	0.490	0.949	0.946	0.941	0.913	0.749	0.721	0.933	0.921
H_0 : CSR $R^2 = 0$		0.022	0.017	0.020	0.017	0.015	0.015	0.022	0.023	0.017	0.015	0.012	0.012	0.013	0.006	0.009	0.009
		N = 256															
Panel B		OLS								GLS							
$H_0 : \gamma_{1,j} = 0$	0.10	0.221	0.223	0.217	0.221	0.223	0.215	0.219	0.207	0.394	0.407	0.394	0.406	0.416	0.404	0.404	0.421
FM	0.05	0.141	0.147	0.141	0.145	0.147	0.144	0.144	0.139	0.305	0.327	0.312	0.320	0.331	0.324	0.325	0.335
	0.01	0.053	0.057	0.051	0.056	0.056	0.060	0.058	0.053	0.178	0.197	0.187	0.194	0.198	0.195	0.196	0.201
$H_0 : \gamma_{1,j} = 0$	0.10	0.076	0.078	0.080	0.074	0.077	0.082	0.084	0.079	0.106	0.116	0.110	0.109	0.092	0.085	0.090	0.076
MR	0.05	0.037	0.037	0.042	0.036	0.036	0.045	0.049	0.041	0.060	0.066	0.063	0.064	0.051	0.048	0.046	0.036
	0.01	0.008	0.008	0.012	0.008	0.010	0.018	0.017	0.012	0.020	0.020	0.021	0.015	0.015	0.014	0.014	0.008
$H_0 : \lambda_{1,j} = 0$	0.10	0.077	0.078	0.083	0.076	0.069	0.058	0.062	0.061	0.113	0.116	0.107	0.122	0.082	0.062	0.063	0.059
MR	0.05	0.038	0.036	0.043	0.038	0.035	0.027	0.030	0.026	0.060	0.062	0.059	0.071	0.040	0.028	0.029	0.025
	0.01	0.010	0.009	0.015	0.010	0.010	0.007	0.008	0.006	0.022	0.022	0.020	0.027	0.009	0.007	0.005	0.006
Average CSR R^2		0.132	0.134	0.135	0.136	0.136	0.133	0.137	0.132	0.039	0.042	0.040	0.042	0.043	0.043	0.043	0.044
Median CSR R^2		0.076	0.077	0.080	0.080	0.079	0.073	0.080	0.073	0.019	0.021	0.019	0.020	0.021	0.021	0.020	0.021
H_0 : CSR $R^2 = 1$		0.021	0.024	0.026	0.023	0.028	0.028	0.025	0.026	0.755	0.740	0.743	0.750	0.704	0.668	0.670	0.655
H_0 : CSR $R^2 = 0$		0.008	0.008	0.012	0.008	0.010	0.018	0.017	0.012	0.026	0.030	0.029	0.026	0.018	0.017	0.014	0.011

Notes: This table reports the probability of rejecting $H_0 : \gamma_{1,j} = 0$ (price of beta risk) and $H_0 : \lambda_{1,j} = 0$ (price of covariance risk) using a two-tailed t-test at various significance levels. Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a heteroskedasticity and autocorrelation consistent variance-covariance matrix with automatic lag selection and a Bartlett kernel (see Newey and West, 1994). The table also reports the average/median cross-sectional R^2 and rejection rates for the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_{1,j} = 0$). Returns are independently drawn from $N(\mu, V)$ where μ and V are set equal to the average and estimated variance-covariance matrix of the actual returns (25 FF size and book-to-market portfolios with the sample period ending in December 2020). For N = 1024 and $j = 1, \ldots, 5, > 5$ we use boundary independent coefficients only. We ignore boundary effects and use circular filtering in the other cases.

Table IA.5: Rejection Rates with Simulated Returns and a Useless Factor: MODWPT MRA

		Beta decomposition based on MODWPT MRA, Indirect															
Frequency in cycles per period		$n=0$ $[0, \frac{1}{16}]$	1 $[\frac{1}{16}, \frac{1}{8}]$	2 $[\frac{1}{8}, \frac{3}{16}]$	3 $[\frac{3}{16}, \frac{1}{4}]$	4 $[\frac{1}{4}, \frac{5}{16}]$	5 $[\frac{5}{16}, \frac{3}{8}]$	6 $[\frac{3}{8}, \frac{7}{16}]$	7 $[\frac{7}{16}, \frac{1}{2}]$	$n=0$ $[0, \frac{1}{16}]$	1 $[\frac{1}{16}, \frac{1}{8}]$	2 $[\frac{1}{8}, \frac{3}{16}]$	3 $[\frac{3}{16}, \frac{1}{4}]$	4 $[\frac{1}{4}, \frac{5}{16}]$	5 $[\frac{5}{16}, \frac{3}{8}]$	6 $[\frac{3}{8}, \frac{7}{16}]$	7 $[\frac{7}{16}, \frac{1}{2}]$
		$N = 1024$															
Panel A		OLS								GLS							
$H_0 : \gamma_{1,n} = 0$	0.10	0.485	0.487	0.493	0.487	0.498	0.498	0.501	0.506	0.425	0.430	0.422	0.425	0.422	0.423	0.425	0.430
FM	0.05	0.400	0.400	0.412	0.411	0.421	0.418	0.416	0.426	0.343	0.347	0.338	0.339	0.339	0.337	0.346	0.350
	0.01	0.265	0.266	0.280	0.276	0.285	0.273	0.281	0.284	0.210	0.205	0.214	0.202	0.212	0.194	0.210	0.210
$H_0 : \gamma_{1,n} = 0$	0.10	0.121	0.126	0.134	0.124	0.130	0.130	0.133	0.135	0.075	0.076	0.082	0.079	0.090	0.076	0.079	0.085
MR	0.05	0.067	0.072	0.078	0.065	0.075	0.074	0.075	0.076	0.038	0.035	0.040	0.036	0.043	0.034	0.036	0.039
	0.01	0.017	0.020	0.018	0.017	0.020	0.015	0.020	0.020	0.006	0.004	0.006	0.006	0.009	0.006	0.005	0.006
$H_0 : \lambda_{1,n} = 0$	0.10	0.120	0.120	0.133	0.125	0.129	0.127	0.132	0.131	0.074	0.078	0.082	0.078	0.086	0.077	0.077	0.080
MR	0.05	0.065	0.069	0.075	0.064	0.073	0.070	0.073	0.073	0.034	0.036	0.039	0.036	0.042	0.034	0.037	0.037
	0.01	0.014	0.018	0.017	0.014	0.021	0.015	0.017	0.015	0.005	0.006	0.007	0.008	0.008	0.006	0.006	0.005
Average CSR R^2		0.134	0.134	0.136	0.132	0.136	0.136	0.138	0.137	0.042	0.041	0.042	0.041	0.043	0.040	0.041	0.042
Median CSR R^2		0.083	0.079	0.080	0.079	0.083	0.081	0.083	0.084	0.020	0.020	0.019	0.019	0.020	0.019	0.020	0.020
H_0 : CSR $R^2 = 1$		0.494	0.489	0.484	0.501	0.481	0.485	0.476	0.477	0.930	0.941	0.937	0.944	0.934	0.943	0.940	0.936
H_0 : CSR $R^2 = 0$		0.017	0.020	0.018	0.017	0.020	0.015	0.020	0.020	0.011	0.012	0.014	0.014	0.017	0.011	0.013	0.012
		$N = 256$															
Panel B		OLS								GLS							
$H_0 : \gamma_{1,n} = 0$	0.10	0.172	0.167	0.174	0.173	0.169	0.169	0.172	0.166	0.346	0.345	0.338	0.340	0.345	0.347	0.332	0.345
FM	0.05	0.108	0.099	0.109	0.102	0.102	0.100	0.100	0.104	0.259	0.260	0.256	0.253	0.261	0.262	0.246	0.255
	0.01	0.033	0.032	0.033	0.032	0.032	0.036	0.032	0.030	0.135	0.139	0.137	0.136	0.138	0.138	0.126	0.134
$H_0 : \gamma_{1,n} = 0$	0.10	0.083	0.088	0.084	0.080	0.085	0.083	0.084	0.092	0.092	0.135	0.135	0.136	0.140	0.136	0.126	0.147
MR	0.05	0.044	0.045	0.047	0.046	0.050	0.047	0.050	0.050	0.049	0.087	0.082	0.082	0.085	0.087	0.078	0.093
	0.01	0.016	0.017	0.015	0.018	0.019	0.020	0.019	0.018	0.019	0.032	0.034	0.029	0.034	0.034	0.032	0.037
$H_0 : \lambda_{1,n} = 0$	0.10	0.077	0.089	0.085	0.084	0.085	0.081	0.085	0.088	0.088	0.133	0.130	0.137	0.139	0.140	0.129	0.135
MR	0.05	0.045	0.051	0.051	0.049	0.047	0.047	0.053	0.048	0.047	0.080	0.078	0.084	0.088	0.083	0.078	0.079
	0.01	0.019	0.024	0.023	0.020	0.022	0.021	0.021	0.021	0.018	0.033	0.031	0.037	0.037	0.034	0.034	0.031
Average CSR R^2		0.142	0.144	0.142	0.142	0.141	0.141	0.143	0.145	0.041	0.042	0.042	0.041	0.041	0.042	0.040	0.041
Median CSR R^2		0.082	0.084	0.083	0.085	0.085	0.084	0.086	0.086	0.020	0.020	0.019	0.020	0.020	0.020	0.020	0.019
H_0 : CSR $R^2 = 1$		0.005	0.002	0.005	0.005	0.003	0.004	0.004	0.005	0.316	0.313	0.327	0.320	0.319	0.318	0.319	0.322
H_0 : CSR $R^2 = 0$		0.016	0.017	0.015	0.018	0.019	0.020	0.019	0.018	0.023	0.041	0.041	0.039	0.040	0.043	0.039	0.045

Notes: This table reports the probability of rejecting $H_0 : \gamma_{1,n} = 0$ (price of beta risk) and $H_0 : \lambda_{1,n} = 0$ (price of covariance risk) for $n = 0, \ldots, 7$ with $j = 3$ using a two-tailed t-test at various significance levels. Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a heteroskedasticity and autocorrelation consistent variance-covariance matrix with automatic lag selection and a Bartlett kernel (see Newey and West, 1994). The table also reports the average/median cross-sectional R^2 and rejection rates for the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_{1,n} = 0$). Returns are independently drawn from $N(\mu, V)$ where μ and V are set equal to the average and estimated variance-covariance matrix of the actual returns (25 FF size and book-to-market portfolios with the sample period ending in December 2020). We use boundary independent coefficients only.

Table IA.6a: Rejection Rates with Simulated Returns, N = 256 : Frequency-specific Factor Priced at High Frequencies

		Beta decomposition based on												
		MODWT MRA, Indirect					MODWPT MRA, Indirect							
Frequency in cycles per period		$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	$n=0$ $[0,\frac{1}{16}]$	1 $[\frac{1}{16},\frac{1}{8}]$	2 $[\frac{1}{8},\frac{3}{16}]$	3 $[\frac{3}{16},\frac{1}{4}]$	4 $[\frac{1}{4},\frac{5}{16}]$	5 $[\frac{5}{16},\frac{3}{8}]$	6 $[\frac{3}{8},\frac{7}{16}]$	7 $[\frac{7}{16},\frac{1}{2}]$
Panel A		OLS estimator												
$H_0:\gamma_1=0$	0.10	1.0000	0.1792	0.1648	0.3330	0.3500	0.2908	0.1648	0.2284	0.2280	0.1046	0.2702	0.0704	1.0000
	0.05	1.0000	0.1270	0.1136	0.2678	0.2784	0.2262	0.1136	0.1700	0.1708	0.0662	0.2018	0.0452	1.0000
	0.01	1.0000	0.0670	0.0586	0.1872	0.1794	0.1428	0.0586	0.0960	0.1052	0.0300	0.1254	0.0206	1.0000
$H_0:\lambda_1=0$	0.10	1.0000	0.1698	0.1308	0.3166	0.3328	0.2562	0.1308	0.1904	0.2242	0.0948	0.2378	0.0532	1.0000
	0.05	1.0000	0.1166	0.0824	0.2506	0.2576	0.1872	0.0824	0.1320	0.1668	0.0582	0.1684	0.0318	1.0000
	0.01	1.0000	0.0558	0.0302	0.1696	0.1534	0.0984	0.0302	0.0642	0.0958	0.0222	0.0906	0.0080	1.0000
Average γ_0		0.0715	2.2039	2.2102	2.1766	2.1977	2.2193	2.2102	2.2143	2.2154	2.2198	2.2064	2.2192	0.0335
Average CSR R^2		96.39%	5.73%	5.23%	5.76%	4.89%	4.72%	5.23%	5.29%	4.78%	4.42%	5.12%	4.77%	97.89%
Median CSR R^2		96.52%	2.96%	2.62%	2.85%	2.47%	2.31%	2.62%	2.61%	2.27%	2.04%	2.41%	2.38%	97.98%
H_0 : CSR $R^2=1$		0.0000	0.9212	0.3702	0.0108	0.0000	0.1756	0.3702	0.7140	0.9084	0.9278	0.9232	0.8212	0.0000
H_0 : CSR $R^2=0$		1.0000	0.0670	0.0586	0.1872	0.1794	0.1428	0.0586	0.0960	0.1052	0.0300	0.1254	0.0206	1.0000
Panel B		GLS estimator												
$H_0:\gamma_1=0$	0.10	1.0000	0.0826	0.0834	0.2606	0.0624	0.1134	0.0834	0.1288	0.1470	0.0674	0.2006	0.0352	1.0000
	0.05	1.0000	0.0492	0.0474	0.1944	0.0326	0.0706	0.0474	0.0846	0.1022	0.0352	0.1418	0.0186	1.0000
	0.01	1.0000	0.0204	0.0206	0.1260	0.0104	0.0364	0.0206	0.0464	0.0496	0.0128	0.0762	0.0074	1.0000
$H_0:\lambda_1=0$	0.10	1.0000	0.0726	0.0482	0.2418	0.0546	0.0898	0.0482	0.1044	0.1344	0.0554	0.2030	0.0374	1.0000
	0.05	1.0000	0.0430	0.0254	0.1874	0.0284	0.0564	0.0254	0.0654	0.0822	0.0302	0.1446	0.0192	1.0000
	0.01	1.0000	0.0200	0.0120	0.1196	0.0092	0.0282	0.0120	0.0302	0.0462	0.0096	0.0786	0.0084	1.0000
Average γ_0		0.0444	0.1716	0.2646	0.1163	0.1161	0.2650	0.2646	0.2652	0.2651	0.2649	0.2655	0.2656	0.0263
Average CSR R^2		65.30%	0.65%	0.79%	1.67%	1.57%	0.75%	0.79%	0.78%	0.77%	0.74%	0.88%	0.79%	85.62%
Median CSR R^2		65.49%	0.29%	0.35%	0.77%	0.71%	0.33%	0.35%	0.33%	0.34%	0.32%	0.39%	0.34%	85.95%
H_0 : CSR $R^2=1$		0.0062	0.9994	0.9560	0.0000	0.0000	0.7582	0.9560	0.9880	0.9912	0.9938	0.9864	0.9906	0.0000
H_0 : CSR $R^2=0$		1.0000	0.0272	0.0262	0.1262	0.0118	0.0420	0.0262	0.0556	0.0654	0.0196	0.0936	0.0078	1.0000

Notes: This table reports the probability of rejecting $H_0:\gamma_1=0$ (price of beta risk) and $H_0:\lambda_1=0$ (price of covariance risk) using a two-tailed t-test at various significance levels. Inference is based on Kan et al. (2013) misspecification robust t-statistics using a heteroskedasticity and autocorrelation consistent variance-covariance matrix with automatic lag selection and a Bartlett kernel (see Newey and West, 1994). The table also reports the average zero-beta rate γ_0, the average/median cross-sectional R^2 and rejection rates for the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0:R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0:\gamma_1=0$). The test assets are 5,000 sets of simulated returns for 25 portfolios with $N=256$ observations using a frequency-specific DGP based on the MODWPT. We use boundary independent coefficients only. The simulated returns offer different risk premia because they are differentially exposed with respect to a useful factor at a frequency-band $f\in[0.469,0.50]$ (i.e., **high frequencies**, $n^\star=15$). The filter used is LA(8) with an OLS estimator in Panel A and a GLS estimator in Panel B.

Table IA.6b: Rejection Rates with Simulated Returns, N = 256 : Frequency-specific Factor Priced at Low Frequencies

		Beta decomposition based on												
		MODWT MRA, Indirect					MODWPT MRA, Indirect							
Frequency in cycles per period		$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	$n=0$ $[0,\frac{1}{16}]$	1 $[\frac{1}{16},\frac{1}{8}]$	2 $[\frac{1}{8},\frac{3}{16}]$	3 $[\frac{3}{16},\frac{1}{4}]$	4 $[\frac{1}{4},\frac{5}{16}]$	5 $[\frac{5}{16},\frac{3}{8}]$	6 $[\frac{3}{8},\frac{7}{16}]$	7 $[\frac{7}{16},\frac{1}{2}]$
Panel C		OLS estimator												
$H_0:\gamma_1=0$	0.10	0.1754	0.2366	0.0288	0.8166	1.0000	1.0000	0.0288	0.1992	0.1976	0.0992	0.1314	0.1242	0.1514
	0.05	0.1152	0.1750	0.0200	0.7360	1.0000	1.0000	0.0200	0.1448	0.1410	0.0568	0.0876	0.0796	0.0890
	0.01	0.0558	0.0936	0.0090	0.4790	0.9994	1.0000	0.0090	0.0774	0.0704	0.0132	0.0404	0.0386	0.0294
$H_0:\lambda_1=0$	0.10	0.1718	0.2272	0.0182	0.6800	1.0000	1.0000	0.0182	0.1832	0.1970	0.0804	0.1298	0.1180	0.1088
	0.05	0.1154	0.1646	0.0088	0.4936	1.0000	1.0000	0.0088	0.1246	0.1412	0.0394	0.0854	0.0728	0.0520
	0.01	0.0536	0.0884	0.0020	0.1794	0.1158	0.8442	0.0020	0.0602	0.0746	0.0070	0.0424	0.0278	0.0104
Average γ_0		2.2682	2.2048	2.0185	0.9599	0.0121	0.0194	2.0185	2.0297	2.0289	2.0302	2.0147	2.0115	2.0300
Average CSR R^2		4.68%	4.96%	5.36%	31.84%	98.99%	98.98%	5.36%	5.30%	4.65%	4.54%	5.52%	5.59%	4.98%
Median CSR R^2		2.37%	2.50%	2.82%	31.61%	99.03%	99.02%	2.82%	2.57%	2.35%	2.22%	2.75%	2.85%	2.51%
H_0 : CSR $R^2=1$		0.9684	0.9018	0.0002	0.0000	0.0000	0.0000	0.0002	0.0020	0.2486	0.0230	0.0062	0.0022	0.0022
H_0 : CSR $R^2=0$		0.0558	0.0936	0.0090	0.4790	0.9994	1.0000	0.0090	0.0774	0.0704	0.0132	0.0404	0.0386	0.0294
Panel D		GLS estimator												
$H_0:\gamma_1=0$	0.10	0.0990	0.1766	0.0400	0.1232	1.0000	1.0000	0.0400	0.1706	0.1786	0.0424	0.0728	0.0496	0.0268
	0.05	0.0620	0.1164	0.0256	0.0880	1.0000	1.0000	0.0256	0.1092	0.1224	0.0188	0.0404	0.0256	0.0092
	0.01	0.0314	0.0514	0.0128	0.0484	1.0000	1.0000	0.0128	0.0484	0.0670	0.0068	0.0182	0.0106	0.0018
$H_0:\lambda_1=0$	0.10	0.0946	0.1758	0.0242	0.2278	1.0000	1.0000	0.0242	0.1374	0.1706	0.0322	0.0754	0.0508	0.0160
	0.05	0.0582	0.1150	0.0156	0.1846	1.0000	1.0000	0.0156	0.0838	0.1200	0.0146	0.0452	0.0282	0.0056
	0.01	0.0288	0.0522	0.0068	0.1208	0.1154	0.8756	0.0068	0.0396	0.0636	0.0054	0.0216	0.0114	0.0008
Average γ_0		0.1779	0.1708	0.1454	0.0823	0.0097	0.0155	0.1454	0.1458	0.1456	0.1456	0.1459	0.1457	0.1457
Average CSR R^2		0.41%	0.43%	0.50%	2.01%	85.92%	88.39%	0.50%	0.49%	0.47%	0.46%	0.57%	0.55%	0.47%
Median CSR R^2		0.18%	0.18%	0.22%	1.17%	86.64%	88.62%	0.22%	0.22%	0.21%	0.20%	0.27%	0.25%	0.21%
H_0 : CSR $R^2=1$		1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
H_0 : CSR $R^2=0$		0.0384	0.0624	0.0122	0.0400	1.0000	1.0000	0.0122	0.0612	0.0860	0.0068	0.0226	0.0122	0.0044

Notes: This table reports the probability of rejecting $H_0:\gamma_1=0$ (price of beta risk) and $H_0:\lambda_1=0$ (price of covariance risk) using a two-tailed t-test at various significance levels. Inference is based on Kan et al. (2013) misspecification robust t-statistics using a heteroskedasticity and autocorrelation consistent variance-covariance matrix with automatic lag selection and a Bartlett kernel (see Newey and West, 1994). The table also reports the average zero-beta rate γ_0, the average/median cross-sectional R^2 and rejection rates for the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0:R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0:\gamma_1=0$). The test assets are 5,000 sets of simulated returns for 25 portfolios with $N=1024$ observations using a frequency-specific DGP based on the MODWPT. We use boundary independent coefficients only. The simulated returns offer different risk premia because they are differentially exposed with respect to a useful factor at a frequency-band $f\in[0,0.031]$ (i.e., **low frequencies**, $n^\star=0$). The filter used is LA(8) with an OLS estimator in Panel C and a GLS estimator in Panel D.

Table IA.6c: Rejection Rates with Simulated Returns, N = 256 : Frequency-specific Factor Priced at Medium Frequencies

		Beta decomposition based on												
		MODWT MRA, Indirect					MODWPT MRA, Indirect							
Frequency in cycles per period		$j = 1$ $[\frac{1}{2}, \frac{1}{4}]$	2 $[\frac{1}{4}, \frac{1}{8}]$	3 $[\frac{1}{8}, \frac{1}{16}]$	4 $[\frac{1}{16}, \frac{1}{32}]$	> 4 $[\frac{1}{32}, 0]$	$n = 0$ $[0, \frac{1}{16}]$	1 $[\frac{1}{16}, \frac{1}{8}]$	2 $[\frac{1}{8}, \frac{3}{16}]$	3 $[\frac{3}{16}, \frac{1}{4}]$	4 $[\frac{1}{4}, \frac{5}{16}]$	5 $[\frac{5}{16}, \frac{3}{8}]$	6 $[\frac{3}{8}, \frac{7}{16}]$	7 $[\frac{7}{16}, \frac{1}{2}]$
Panel E		OLS estimator												
$H_0 : \gamma_1 = 0$	0.10	1.0000	1.0000	0.1246	0.1660	0.4198	0.3028	0.1246	0.2634	1.0000	1.0000	0.0678	0.2564	0.2910
	0.05	1.0000	1.0000	0.0764	0.1100	0.3482	0.2354	0.0764	0.2024	1.0000	1.0000	0.0372	0.1942	0.2224
	0.01	1.0000	1.0000	0.0264	0.0482	0.2506	0.1494	0.0264	0.1242	1.0000	1.0000	0.0130	0.1164	0.1352
$H_0 : \lambda_1 = 0$	0.10	1.0000	1.0000	0.1004	0.1478	0.3904	0.2724	0.1004	0.2204	1.0000	1.0000	0.0562	0.2340	0.2768
	0.05	1.0000	1.0000	0.0554	0.0898	0.3172	0.2020	0.0554	0.1562	1.0000	1.0000	0.0296	0.1694	0.1976
	0.01	1.0000	1.0000	0.0114	0.0370	0.2130	0.1096	0.0114	0.0824	1.0000	1.0000	0.0074	0.0920	0.1102
Average γ_0		0.2035	0.0876	2.2175	2.2169	2.2067	2.2147	2.2175	2.2148	0.0499	0.0570	2.2152	2.2176	2.2133
Average CSR R^2		90.23%	95.48%	4.95%	4.92%	5.28%	4.90%	4.95%	5.35%	97.14%	96.85%	4.82%	4.72%	5.02%
Median CSR R^2		90.55%	95.62%	2.45%	2.48%	2.69%	2.46%	2.45%	2.65%	97.22%	96.95%	2.37%	2.32%	2.50%
H_0 : CSR $R^2 = 1$		0.0004	0.0000	0.3724	0.0144	0.0000	0.1650	0.3724	0.7704	0.0004	0.0000	0.8880	0.8692	0.7178
H_0 : CSR $R^2 = 0$		1.0000	1.0000	0.0264	0.0482	0.2506	0.1494	0.0264	0.1242	1.0000	1.0000	0.0130	0.1164	0.1352
Panel F		GLS estimator												
$H_0 : \gamma_1 = 0$	0.10	1.0000	1.0000	0.0636	0.2940	0.2222	0.1382	0.0636	0.1652	1.0000	1.0000	0.0370	0.1878	0.1922
	0.05	1.0000	1.0000	0.0318	0.2268	0.1586	0.0906	0.0318	0.1140	1.0000	1.0000	0.0182	0.1314	0.1342
	0.01	0.9998	1.0000	0.0080	0.1498	0.0852	0.0446	0.0080	0.0652	1.0000	1.0000	0.0046	0.0744	0.0794
$H_0 : \lambda_1 = 0$	0.10	1.0000	1.0000	0.0460	0.2722	0.1946	0.1138	0.0460	0.1366	1.0000	1.0000	0.0306	0.1714	0.1688
	0.05	1.0000	1.0000	0.0192	0.2044	0.1288	0.0738	0.0192	0.0864	1.0000	1.0000	0.0128	0.1170	0.1198
	0.01	0.9996	1.0000	0.0048	0.1234	0.0576	0.0378	0.0048	0.0426	1.0000	1.0000	0.0036	0.0666	0.0652
Average γ_0		0.1117	0.0660	0.2509	0.2976	0.2964	0.2508	0.2509	0.2515	0.0446	0.0487	0.2518	0.2518	0.2510
Average CSR R^2		48.03%	68.84%	1.06%	2.29%	2.22%	1.06%	1.06%	1.09%	77.80%	75.88%	1.14%	1.13%	1.12%
Median CSR R^2		47.83%	69.07%	0.48%	1.07%	1.03%	0.46%	0.48%	0.50%	78.27%	76.40%	0.53%	0.53%	0.49%
H_0 : CSR $R^2 = 1$		0.0844	0.0130	0.9378	0.0034	0.0000	0.6724	0.9378	0.9830	0.0066	0.0082	0.9802	0.9814	0.9546
H_0 : CSR $R^2 = 0$		1.0000	1.0000	0.0120	0.1494	0.0850	0.0552	0.0120	0.0752	1.0000	1.0000	0.0064	0.0880	0.0910

Notes: This table reports the probability of rejecting $H_0 : \gamma_1 = 0$ (price of beta risk) and $H_0 : \lambda_1 = 0$ (price of covariance risk) using a two-tailed t-test at various significance levels. Inference is based on Kan et al. (2013) misspecification robust t-statistics using a heteroskedasticity and autocorrelation consistent variance-covariance matrix with automatic lag selection and a Bartlett kernel (see Newey and West, 1994). The table also reports the average zero-beta rate γ_0, the average/median cross-sectional R^2 and rejection rates for the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_1 = 0$). The test assets are 5,000 sets of simulated returns for 25 portfolios with $N = 1024$ observations using a frequency-specific DGP based on the MODWPT. We use boundary independent coefficients only. The simulated returns offer different risk premia because they are differentially exposed with respect to a useful factor at a frequency-band $f \in [0.219, 0.250]$ (i.e., **medium frequencies**, $n^\star = 7$). The filter used is LA(8) with an OLS estimator in Panel E and a GLS estimator in Panel F.

Table IA.7: Fama-MacBeth Rejection Rates with the Direct Approach and a Useless Factor

		Beta decomposition based on MODWT, Direct													
Frequency in cycles per period		$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	5 $[\frac{1}{32},\frac{1}{64}]$	>5 $[\frac{1}{64},0]$	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	5 $[\frac{1}{32},\frac{1}{64}]$	>5 $[\frac{1}{64},0]$
Panel A		OLS							GLS						
		N = 1024							**N = 1024**						
$H_0:\gamma_{1,j}=0$	0.10	0.4214	0.4134	0.5112	0.4516	0.5292	0.4982	0.4282	0.3686	0.3452	0.4222	0.3188	0.3686	0.3612	0.4188
	0.05	0.3382	0.3276	0.4370	0.3710	0.4550	0.4024	0.3478	0.2812	0.2592	0.3330	0.2320	0.2816	0.2682	0.3292
	0.01	0.2126	0.1946	0.2952	0.2440	0.3106	0.2454	0.2172	0.1520	0.1300	0.1984	0.1058	0.1522	0.1364	0.1884
Average CSR R^2		0.1124	0.1242	0.1459	0.1388	0.1744	0.1694	0.1443	0.0412	0.0391	0.0470	0.0344	0.0427	0.0387	0.0496
		N = 256							**N = 256**						
$H_0:\gamma_{1,j}=0$	0.10	0.0668	0.0400	0.0604	0.0608	0.0724	0.0176	0.1484	0.3286	0.2548	0.2800	0.2834	0.3990	0.2284	0.5012
	0.05	0.0256	0.0158	0.0290	0.0264	0.0346	0.0080	0.0776	0.2334	0.1612	0.2026	0.1938	0.3092	0.1448	0.4108
	0.01	0.0032	0.0026	0.0048	0.0024	0.0072	0.0008	0.0210	0.1102	0.0640	0.0910	0.0830	0.1668	0.0454	0.2604
Average CSR R^2		0.1277	0.1123	0.0819	0.1060	0.0992	0.0524	0.1460	0.0452	0.0338	0.0386	0.0380	0.0561	0.0304	0.0743
		N = 128							**N = 128**						
$H_0:\gamma_{1,j}=0$	0.10	0.2778	0.2806	0.4268	0.2868	0.5800	0.3278	0.8276	0.2600	0.2510	0.2004	0.4022	0.6158	0.4642	0.7390
	0.05	0.1484	0.1732	0.2442	0.1174	0.4494	0.2346	0.7490	0.1768	0.1658	0.1150	0.3038	0.5412	0.3808	0.6828
	0.01	0.0138	0.0190	0.0118	0.0010	0.0438	0.0324	0.0654	0.0732	0.0634	0.0282	0.1558	0.3970	0.2348	0.5698
Average CSR R^2		0.1436	0.1448	0.2160	0.1472	0.3418	0.2084	0.4712	0.0377	0.0351	0.0280	0.0565	0.1174	0.0748	0.1659
Panel B		OLS							GLS						
		N = 1024							**N = 1024**						
$H_0:\gamma_{1,j}=0$	0.10	0.5192	0.5028	0.5068	0.4942	0.4904	0.4758	0.4756	0.4640	0.4382	0.4358	0.4280	0.4314	0.4108	0.4166
	0.05	0.4418	0.4276	0.4256	0.4112	0.4070	0.3988	0.3940	0.3820	0.3580	0.3538	0.3458	0.3448	0.3294	0.3338
	0.01	0.3100	0.2902	0.2896	0.2700	0.2750	0.2562	0.2492	0.2506	0.2268	0.2190	0.2102	0.2130	0.2008	0.2010
Average CSR R^2		0.1381	0.1349	0.1349	0.1345	0.1341	0.1387	0.1358	0.0446	0.0420	0.0408	0.0416	0.0419	0.0426	0.0433
		N = 256							**N = 256**						
$H_0:\gamma_{1,j}=0$	0.10	0.2146	0.2180	0.2226	0.2214	0.2192	0.2206	0.2170	0.3958	0.4154	0.4088	0.3884	0.4182	0.4196	0.4094
	0.05	0.1342	0.1426	0.1452	0.1452	0.1458	0.1474	0.1390	0.3128	0.3362	0.3242	0.3086	0.3334	0.3294	0.3252
	0.01	0.0504	0.0550	0.0536	0.0542	0.0542	0.0572	0.0518	0.1862	0.1964	0.1966	0.1862	0.2036	0.1976	0.1976
Average CSR R^2		0.1316	0.1342	0.1360	0.1365	0.1367	0.1341	0.1367	0.0403	0.0424	0.0421	0.0409	0.0435	0.0427	0.0432
		N = 128							**N = 128**						
$H_0:\gamma_{1,j}=0$	0.10	0.3772	0.3818	0.3732	0.3748	0.3784	0.3796	0.3760	0.4214	0.4202	0.4160	0.4180	0.4224	0.4222	0.4282
	0.05	0.2796	0.2896	0.2688	0.2834	0.2866	0.2826	0.2792	0.3302	0.3380	0.3330	0.3364	0.3392	0.3416	0.3430
	0.01	0.1360	0.1426	0.1302	0.1386	0.1370	0.1356	0.1390	0.2014	0.2144	0.2020	0.2098	0.2136	0.2098	0.2116
Average CSR R^2		0.1507	0.1556	0.1514	0.1513	0.1550	0.1524	0.1568	0.0411	0.0425	0.0415	0.0406	0.0419	0.0415	0.0420

Notes: This table reports the probability of rejecting $H_0:\gamma_{1,j}=0$ (price of beta risk) using a two-tailed t-test at various significance levels. Inference is based on Fama-MacBeth (1973) t-statistics. The table also reports the average cross-sectional R^2. In Panel A the test assets are actual returns (the 25 Fama-French size and book-to-market value-weighted portfolios with the sample period ending in December 2020). In Panel B the test assets are simulated returns, independently drawn from $N(\mu, V)$ where μ and V are set equal to the average and estimated variance-covariance matrix of the actual returns. The beta decomposition is based on the MODWT direct method using only boundary independent coefficients for $N=1024$. For $N=256,128$ we ignore boundary effects and use circular filtering. The filter used is LA(8).

Table IA.8: Spectral Zoo: Macro Uncertainty Index from Xyngis (2017)

size and investment portfolios

Panel A		Beta decomposition based on MODWT MRA, Indirect							
Frequency in cycles per period		$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	5 $[\frac{1}{32},\frac{1}{64}]$	6 $[\frac{1}{64},\frac{1}{128}]$	>6 $[\frac{1}{128},0]$	>5 $[\frac{1}{64},0]$
OLS	$\gamma_{0,j}$	0.5336	0.6802	0.5134	0.3292	0.3349	0.5368	0.7525	0.5642
	FM t-statistic	2.8324	3.2154	2.7759	1.3894	1.5903	2.8574	3.7837	2.8234
	MR t-statistic	1.7963	1.9703	0.9899	0.7222	1.1704	2.2119	3.3844	2.0818
	boot p-val	0.2290	0.0750	0.6100	0.6430	0.4145	0.0425	0.0020	0.0555
OLS	$\gamma_{1,j}$	-0.2056	-0.0703	-0.4215	-0.4140	-0.2653	-0.2422	-0.0942	-0.3156
	FM t-statistic	-0.9737	-0.2763	-1.0379	-1.4124	-1.7816	-3.0402	-1.5939	-3.7330
	MR t-statistic	-0.7161	-0.2015	-0.4694	-0.8148	-1.3089	-1.6732	-0.8772	-1.9149
	boot p-val	0.6365	0.8695	0.9045	0.6420	0.3785	0.1555	0.5125	0.0980
GLS	$\gamma_{1,j}$	0.0002	0.0531	-0.2450	-0.3463	-0.2860	-0.0711	-0.0833	-0.1359
	FM t-statistic	0.0022	0.5218	-1.5496	-2.7599	-3.6564	-1.3863	-2.7308	-2.2457
	MR t-statistic	0.0014	0.3431	-0.8262	-1.0206	-1.5115	-0.7372	-1.3672	-1.1704
	boot p-val	0.9995	0.7125	0.8915	0.8115	0.5845	0.4940	0.3050	0.3155
OLS	CSR R^2	6.861%	0.589%	6.431%	16.877%	30.187%	60.176%	15.405%	62.569%
	$se\left(\widehat{R^2}\right)$	0.1949	0.0553	0.2631	0.4023	0.3512	0.2848	0.3495	0.2070
GLS	CSR R^2	0.000%	0.298%	2.626%	8.332%	14.624%	2.102%	8.157%	5.516%
	$se\left(\widehat{R^2}\right)$	0.0001	0.0136	0.0641	0.1623	0.1627	0.0574	0.1118	0.0876
GLS	H_0 : CSR $R^2=1$	0.0014	0.0008	0.0029	0.0146	0.0349	0.0020	0.0084	0.0049
	H_0 : CSR $R^2=0$	0.9989	0.7280	0.3986	0.2971	0.0693	0.4561	0.1406	0.2108

Panel B		Beta decomposition based on MODWPT MRA, Indirect							
Frequency in cycles per period		$n=0$ $[0,\frac{1}{16}]$	1 $[\frac{1}{16},\frac{1}{8}]$	2 $[\frac{1}{8},\frac{3}{16}]$	3 $[\frac{3}{16},\frac{1}{4}]$	4 $[\frac{1}{4},\frac{5}{16}]$	5 $[\frac{5}{16},\frac{3}{8}]$	6 $[\frac{3}{8},\frac{7}{16}]$	7 $[\frac{7}{16},\frac{1}{2}]$
OLS	$\gamma_{0,n}$	0.2493	0.9421	0.8793	0.7058	0.5426	0.8893	0.6840	0.7323
	MR t-statistic	0.5242	4.7965	4.1738	1.3065	1.0079	3.1982	2.3606	3.5275
	boot p-val	0.6875	0.0005	0.0000	0.2655	0.2735	0.0020	0.0065	0.0020
OLS	$\gamma_{1,n}$	-0.4194	0.4820	0.2307	0.0100	-0.1216	0.2505	0.1889	0.0367
	MR t-statistic	-0.7402	1.5450	0.7184	0.0421	-0.3136	1.1746	0.7866	0.2807
	boot p-val	0.5655	0.2205	0.4780	0.9680	0.6925	0.5245	0.3740	0.8045
GLS	$\gamma_{1,n}$	-0.0915	0.2233	0.0260	0.0264	0.0447	0.0105	0.0487	0.0509
	MR t-statistic	-0.3036	1.0562	0.1475	0.4233	0.3737	0.1397	1.0371	0.7380
	boot p-val	0.7725	0.3045	0.8730	0.5295	0.5735	0.8630	0.2705	0.3880
OLS	CSR R^2	13.322%	16.905%	8.519%	0.029%	2.063%	14.035%	10.541%	1.491%
	$se\left(\widehat{R^2}\right)$	0.3053	0.1637	0.1825	0.0137	0.1079	0.2738	0.2718	0.1041
GLS	CSR R^2	0.443%	3.925%	0.109%	0.345%	0.715%	0.051%	2.622%	2.298%
	$se\left(\widehat{R^2}\right)$	0.0289	0.0564	0.0145	0.0155	0.0231	0.0075	0.0500	0.0462
GLS	H_0 : CSR $R^2=1$	0.0008	0.0006	0.0002	0.0008	0.0011	0.0006	0.0006	0.0007
	H_0 : CSR $R^2=0$	0.7619	0.2487	0.8826	0.6633	0.6586	0.8886	0.2840	0.4482

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) macro uncertainty index (innovations: first-difference, $h=1$, the factor is priced at $j=6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0: R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0: \gamma_1=0$) for the GLS case. The test assets are the 25 FF size and investment value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.9: Spectral Zoo: Macro Uncertainty Index from Xyngis (2017)

size and operating profitability portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.2389	0.2275	0.2587	0.1749	0.4273	0.5216	0.7251	0.5473
	FM t-statistic	1.2853	1.1141	1.4350	0.8327	2.2461	2.8496	3.6381	2.8279
	MR t-statistic	0.7541	0.7735	0.8538	0.5047	1.5189	2.2332	3.4695	2.1760
	boot p-val	0.5760	0.5030	0.5560	0.6515	0.2915	0.0360	0.0025	0.0355
OLS	$\gamma_{1,j}$	-0.4316	-0.4369	-0.7855	-0.5109	-0.1859	-0.2307	-0.0294	-0.3028
	FM t-statistic	-2.3682	-1.9993	-2.9960	-2.6546	-1.4737	-2.4817	-0.4832	-2.8613
	MR t-statistic	-1.1620	-1.5058	-1.9546	-1.5945	-1.0169	-1.2770	-0.2890	-1.3713
	boot p-val	0.3190	0.2395	0.3410	0.3165	0.4195	0.2605	0.8130	0.2315
GLS	$\gamma_{1,j}$	0.0192	0.1796	-0.0619	-0.0437	-0.0454	-0.0613	0.0096	-0.0546
	FM t-statistic	0.2178	1.8678	-0.4398	-0.4155	-0.7332	-1.0995	0.2828	-0.7790
	MR t-statistic	0.1441	1.0309	-0.3043	-0.2393	-0.3823	-0.6876	0.1655	-0.5244
	boot p-val	0.8855	0.3740	0.7780	0.8455	0.7290	0.4920	0.8615	0.5975
OLS	CSR R^2	41.153%	25.639%	45.349%	50.981%	19.054%	51.485%	1.476%	47.563%
	$se\left(\widehat{R^2}\right)$	0.2275	0.2097	0.3629	0.3763	0.3453	0.2707	0.1218	0.2209
GLS	CSR R^2	0.112%	8.272%	0.459%	0.409%	1.275%	2.867%	0.190%	1.439%
	$se\left(\widehat{R^2}\right)$	0.0147	0.1556	0.0290	0.0334	0.0601	0.0738	0.0225	0.0521
GLS	H_0 : CSR $R^2 = 1$	0.0940	0.1637	0.0851	0.1125	0.1084	0.1079	0.0647	0.1019
	H_0 : CSR $R^2 = 0$	0.8824	0.2913	0.7620	0.8090	0.6899	0.4787	0.8687	0.5837

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.1769	0.4696	0.4781	0.0712	0.4343	0.6850	0.6490	0.4759
	MR t-statistic	0.4077	1.4109	1.7307	0.1179	0.4465	2.8925	3.7131	1.9340
	boot p-val	0.7445	0.1265	0.0730	0.8970	0.6095	0.0030	0.0010	0.1235
OLS	$\gamma_{1,n}$	-0.4580	-0.3341	-0.2043	-0.2474	-0.1761	0.0461	-0.0113	-0.1250
	MR t-statistic	-0.9213	-0.5707	-0.5954	-0.8840	-0.2448	0.1835	-0.0540	-0.8473
	boot p-val	0.4670	0.5775	0.5750	0.4470	0.7795	0.8390	0.9500	0.4845
GLS	$\gamma_{1,n}$	-0.0182	0.0395	0.0922	0.0943	0.1191	0.0369	0.0277	-0.1389
	MR t-statistic	-0.0759	0.2644	0.7347	0.9622	1.4160	0.5846	0.7426	-1.3271
	boot p-val	0.9400	0.7600	0.5215	0.6785	0.4845	0.5950	0.3315	0.6580
OLS	CSR R^2	24.652%	9.927%	7.248%	16.482%	3.853%	0.777%	0.048%	16.164%
	$se\left(\widehat{R^2}\right)$	0.4874	0.3135	0.2721	0.3371	0.2984	0.0883	0.0151	0.3033
GLS	CSR R^2	0.054%	0.352%	4.547%	7.137%	10.079%	1.874%	1.448%	15.226%
	$se\left(\widehat{R^2}\right)$	0.0115	0.0265	0.1267	0.1414	0.1306	0.0594	0.0319	0.1865
GLS	H_0 : CSR $R^2 = 1$	0.0914	0.0751	0.1206	0.2262	0.3069	0.1193	0.0755	0.5269
	H_0 : CSR $R^2 = 0$	0.9390	0.7920	0.4628	0.3035	0.1365	0.5644	0.4501	0.1353

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) macro uncertainty index (innovations: first-difference, $h = 1$, the factor is priced at $j = 6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_1 = 0$) for the GLS case. The test assets are the 25 FF size and operating profitability value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0 = 3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.10: Spectral Zoo: Industrial Production Growth from Boons and Tamoni (2015)

size and investment portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.4732	0.6927	0.5119	0.7724	0.6588	0.2504	0.9190	0.3292
	FM t-statistic	2.5185	3.7343	2.5958	4.2866	3.1332	1.1748	5.0696	1.9198
	MR t-statistic	1.7314	2.7465	1.8951	3.3555	2.5439	0.5193	5.0320	0.7030
	boot p-val	0.1155	0.0155	0.1370	0.0105	0.0320	0.7490	0.0000	0.6390
OLS	$\gamma_{1,j}$	-0.6068	0.3463	0.0747	-0.0010	0.0035	0.0221	-0.0043	0.0198
	FM t-statistic	-2.2192	2.3637	1.1133	-0.0591	0.3944	3.8014	-2.7903	2.6671
	MR t-statistic	-1.4618	1.3134	0.8600	-0.0457	0.3289	2.1678	-1.4978	0.9870
	boot p-val	0.3495	0.3875	0.6770	0.9660	0.7725	0.3465	0.3685	0.6315
GLS	$\gamma_{1,j}$	0.0125	0.1681	0.0598	-0.0131	-0.0011	0.0049	-0.0015	0.0022
	FM t-statistic	0.0890	2.7943	2.1462	-1.3734	-0.2077	1.4149	-1.5068	0.5516
	MR t-statistic	0.0462	1.4889	1.1962	-0.5005	-0.1088	0.5941	-0.7631	0.2189
	boot p-val	0.9735	0.2150	0.8555	0.7760	0.8970	0.6290	0.6750	0.8180
OLS	CSR R^2	13.251%	38.399%	12.989%	0.035%	1.607%	50.504%	28.375%	19.395%
	$se\left(\widehat{R^2}\right)$	0.2009	0.2924	0.2531	0.0154	0.0968	0.2991	0.3531	0.4635
GLS	CSR R^2	0.009%	8.541%	5.038%	2.063%	0.047%	2.190%	2.484%	0.333%
	$se\left(\widehat{R^2}\right)$	0.0038	0.0984	0.0928	0.0786	0.0073	0.0775	0.0683	0.0293
GLS	H_0 : CSR $R^2=1$	0.0014	0.0011	0.0046	0.0007	0.0011	0.0077	0.0003	0.0027
	H_0 : CSR $R^2=0$	0.9630	0.0672	0.2613	0.6061	0.9138	0.5768	0.4812	0.8265

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.8871	0.6244	1.2069	1.1520	0.6877	0.6862	0.6775	0.7592
	MR t-statistic	2.8981	3.3083	2.7401	2.1727	1.4185	2.7875	3.1405	2.3409
	boot p-val	0.0080	0.0115	0.0470	0.0785	0.1860	0.0035	0.0010	0.0140
OLS	$\gamma_{1,n}$	-0.0140	0.0141	0.2222	0.2233	0.0019	-0.0108	-0.1724	0.2319
	MR t-statistic	-0.5535	0.3584	2.1713	0.7163	0.0083	-0.0239	-0.6926	0.6686
	boot p-val	0.5380	0.7190	0.3515	0.5425	0.9940	0.9845	0.4270	0.5510
GLS	$\gamma_{1,n}$	-0.0377	-0.0380	0.0642	0.1267	0.0722	-0.0153	0.0868	0.2505
	MR t-statistic	-1.6029	-1.1778	0.9149	0.9683	0.5317	-0.1665	0.6919	1.4124
	boot p-val	0.0900	0.2110	0.5260	0.6015	0.6650	0.8630	0.4955	0.1895
OLS	CSR R^2	3.866%	1.493%	42.689%	20.104%	0.001%	0.013%	8.363%	6.414%
	$se\left(\widehat{R^2}\right)$	0.1363	0.0808	0.2926	0.4750	0.0036	0.0103	0.1952	0.1898
GLS	CSR R^2	10.999%	3.301%	4.161%	6.821%	1.690%	0.073%	2.607%	9.945%
	$se\left(\widehat{R^2}\right)$	0.1241	0.0485	0.0880	0.1200	0.0577	0.0093	0.0685	0.1280
GLS	H_0 : CSR $R^2=1$	0.0018	0.0004	0.0022	0.0019	0.0003	0.0007	0.0011	0.0048
	H_0 : CSR $R^2=0$	0.0703	0.2137	0.3747	0.2624	0.5685	0.8674	0.4763	0.1299

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for IPG (innovations: first-difference, the factor in Boons and Tamoni (2015) is priced at $j>4$ with quarterly data which corresponds to $j=6$, >6 or >5). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0:R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0:\gamma_1=0$) for the GLS case. The test assets are the 25 FF size and investment value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only BI coefficients). In both cases the filter is LA(8).

Table IA.11: Spectral Zoo: Industrial Production Growth from Boons and Tamoni (2015)

size and operating profitability portfolios

Panel A		Beta decomposition based on MODWT MRA, Indirect							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.5008	0.6390	0.3575	0.4369	0.4236	0.3686	0.9045	0.4500
	FM t-statistic	2.6079	3.3935	1.9123	2.2136	2.0969	1.9308	4.3886	2.7019
	MR t-statistic	1.6198	2.1634	1.4798	1.3815	1.3761	0.9563	3.7052	0.7605
	boot p-val	0.1075	0.0865	0.1985	0.3140	0.2995	0.4525	0.0015	0.4890
OLS	$\gamma_{1,j}$	-0.4792	0.4044	0.1111	0.0211	0.0105	0.0158	-0.0048	0.0129
	FM t-statistic	-1.4304	2.8202	2.2560	1.2292	1.3492	3.1077	-2.6668	2.4625
	MR t-statistic	-0.8199	1.5799	1.8079	0.7969	1.0103	1.5985	-1.4227	0.5971
	boot p-val	0.6070	0.6130	0.4475	0.5155	0.3645	0.3530	0.4085	0.6275
GLS	$\gamma_{1,j}$	0.3465	0.1276	-0.0055	-0.0129	-0.0013	0.0006	-0.0008	-0.0003
	FM t-statistic	2.5726	2.1036	-0.1977	-1.4982	-0.3140	0.2246	-0.7391	-0.0770
	MR t-statistic	1.5707	1.5051	-0.1380	-1.0993	-0.2341	0.1332	-0.3368	-0.0407
	boot p-val	0.1875	0.5035	0.8980	0.1770	0.7705	0.8855	0.7580	0.9680
OLS	CSR R^2	9.924%	58.558%	46.344%	14.202%	17.428%	35.464%	32.451%	12.091%
	$se\left(\widehat{R^2}\right)$	0.2064	0.2600	0.2818	0.2940	0.2671	0.4822	0.4966	0.4571
GLS	CSR R^2	15.693%	10.493%	0.093%	5.323%	0.234%	0.120%	1.295%	0.014%
	$se\left(\widehat{R^2}\right)$	0.1548	0.1209	0.0123	0.0957	0.0170	0.0173	0.0794	0.0068
GLS	H_0 : CSR $R^2=1$	0.2573	0.1268	0.0766	0.1255	0.0744	0.0780	0.0184	0.0564
	H_0 : CSR $R^2=0$	0.0484	0.0932	0.8900	0.2645	0.8151	0.8927	0.7500	0.9678

Panel B		Beta decomposition based on MODWPT MRA, Indirect							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.3453	0.4183	1.0021	0.9045	0.4091	0.5601	0.6506	0.7581
	MR t-statistic	0.8881	1.7444	3.4211	2.3459	1.0565	1.7286	2.5292	2.7956
	boot p-val	0.5060	0.1070	0.0015	0.0380	0.3180	0.0690	0.0015	0.0070
OLS	$\gamma_{1,n}$	0.0207	0.0559	0.1490	0.1200	-0.1059	0.3500	-0.1364	0.3296
	MR t-statistic	0.6918	1.4697	1.8887	0.6377	-0.6739	1.1389	-0.9988	0.8669
	boot p-val	0.5670	0.2670	0.1865	0.4925	0.5535	0.2815	0.2955	0.4455
GLS	$\gamma_{1,n}$	-0.0114	0.0036	0.0228	0.0466	0.0474	0.1359	0.0356	0.2548
	MR t-statistic	-0.8003	0.0921	0.5059	0.9335	0.4912	1.2396	0.6611	1.3678
	boot p-val	0.3135	0.9380	0.6315	0.3425	0.6260	0.3990	0.5170	0.3750
OLS	CSR R^2	9.303%	34.535%	40.656%	10.487%	7.015%	24.713%	11.971%	13.847%
	$se\left(\widehat{R^2}\right)$	0.2558	0.3284	0.5272	0.2782	0.2141	0.4458	0.1936	0.3212
GLS	CSR R^2	3.053%	0.068%	1.594%	4.353%	1.975%	12.246%	1.702%	16.864%
	$se\left(\widehat{R^2}\right)$	0.0790	0.0145	0.0647	0.0907	0.0772	0.1868	0.0460	0.2201
GLS	H_0 : CSR $R^2=1$	0.0755	0.0721	0.1059	0.1237	0.1247	0.2699	0.1103	0.4058
	H_0 : CSR $R^2=0$	0.4297	0.9266	0.6228	0.3762	0.6138	0.2111	0.5110	0.1610

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for IPG (innovations: first-difference, the factor in Boons and Tamoni (2015) is priced at $j>4$ with quarterly data which corresponds to $j=6$, >6 or >5). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0: R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0:\gamma_1=0$) for the GLS case. The test assets are the 25 FF size and operating profitability value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only BI coefficients). In both cases the filter is LA(8).

Table IA.12: Spectral Zoo: Volatility of IPG from Boons and Tamoni (2015)

size and investment portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j = 1$ $\left[\frac{1}{2}, \frac{1}{4}\right]$	2 $\left[\frac{1}{4}, \frac{1}{8}\right]$	3 $\left[\frac{1}{8}, \frac{1}{16}\right]$	4 $\left[\frac{1}{16}, \frac{1}{32}\right]$	5 $\left[\frac{1}{32}, \frac{1}{64}\right]$	6 $\left[\frac{1}{64}, \frac{1}{128}\right]$	> 6 $\left[\frac{1}{128}, 0\right]$	> 5 $\left[\frac{1}{64}, 0\right]$
OLS	$\gamma_{0,j}$	0.7518	0.5622	0.6812	0.3852	0.2849	0.3063	0.5836	0.3708
	FM t-statistic	3.9473	3.2387	3.9211	2.0354	1.3874	1.5327	3.4652	2.0050
	MR t-statistic	5.2470	1.6746	4.2523	1.2275	0.9153	0.8590	3.2159	1.2484
	boot p-val	0.0005	0.3210	0.0025	0.8175	0.7920	0.5400	0.0080	0.2700
OLS	$\gamma_{1,j}$	0.0352	0.1083	-0.0419	-0.0351	-0.0164	-0.0103	-0.0023	-0.0094
	FM t-statistic	0.6148	2.3265	-1.1629	-1.7170	-2.0696	-3.7323	-1.8365	-2.8917
	MR t-statistic	0.2347	1.5737	-0.6609	-1.3490	-1.4577	-1.5599	-1.2722	-1.4680
	boot p-val	0.8065	0.3960	0.9250	0.8245	0.8285	0.4685	0.2455	0.3875
GLS	$\gamma_{1,j}$	-0.0421	0.0206	0.0144	-0.0182	-0.0149	-0.0048	-0.0007	-0.0047
	FM t-statistic	-1.2476	0.9585	1.0178	-2.0700	-4.5093	-2.9896	-1.2580	-2.7202
	MR t-statistic	-0.9506	0.6303	0.6704	-1.0073	-1.9144	-1.3624	-0.6843	-1.5251
	boot p-val	0.4475	0.4825	0.4770	0.8345	0.7815	0.5995	0.5660	0.5120
OLS	CSR R^2	0.594%	29.345%	5.108%	19.650%	33.303%	62.950%	34.532%	60.159%
	$se\left(\widehat{R^2}\right)$	0.0498	0.2541	0.1499	0.4188	0.4150	0.1676	0.3330	0.3034
GLS	CSR R^2	1.703%	1.005%	1.133%	4.687%	22.242%	9.777%	1.731%	8.094%
	$se\left(\widehat{R^2}\right)$	0.0320	0.0311	0.0215	0.1053	0.1753	0.1170	0.0483	0.0815
GLS	H_0 : CSR $R^2 = 1$	0.0018	0.0011	0.0009	0.0044	0.0666	0.0106	0.0012	0.0036
	H_0 : CSR $R^2 = 0$	0.3322	0.5055	0.4982	0.3581	0.0301	0.1627	0.4553	0.1228

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n = 0$ $\left[0, \frac{1}{16}\right]$	1 $\left[\frac{1}{16}, \frac{1}{8}\right]$	2 $\left[\frac{1}{8}, \frac{3}{16}\right]$	3 $\left[\frac{3}{16}, \frac{1}{4}\right]$	4 $\left[\frac{1}{4}, \frac{5}{16}\right]$	5 $\left[\frac{5}{16}, \frac{3}{8}\right]$	6 $\left[\frac{3}{8}, \frac{7}{16}\right]$	7 $\left[\frac{7}{16}, \frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	1.0828	0.8633	0.8450	1.1657	0.9215	0.6099	0.6862	0.6939
	MR t-statistic	3.6926	3.7726	3.3848	1.7869	1.7378	3.1044	3.4005	3.9184
	boot p-val	0.0030	0.0020	0.0010	0.1225	0.0940	0.0090	0.0035	0.0010
OLS	$\gamma_{1,n}$	0.0148	0.0357	0.0218	0.0559	0.0271	0.0521	0.0040	-0.0158
	MR t-statistic	1.0676	1.2768	0.7048	1.4223	0.6594	0.9521	0.0312	-0.1809
	boot p-val	0.2365	0.3475	0.5310	0.2580	0.5365	0.3665	0.9765	0.8690
GLS	$\gamma_{1,n}$	0.0044	0.0168	0.0260	0.0017	-0.0093	-0.0064	0.0018	-0.0135
	MR t-statistic	0.4770	1.3977	1.8151	0.0844	-0.6123	-0.4747	0.0652	-0.5493
	boot p-val	0.6145	0.3075	0.1745	0.9120	0.4700	0.6205	0.9555	0.5660
OLS	CSR R^2	12.299%	26.285%	9.634%	24.114%	7.136%	10.330%	0.039%	0.804%
	$se\left(\widehat{R^2}\right)$	0.2214	0.2255	0.2470	0.3129	0.2251	0.2056	0.0232	0.0887
GLS	CSR R^2	0.927%	5.524%	11.208%	0.026%	0.743%	0.371%	0.028%	1.411%
	$se\left(\widehat{R^2}\right)$	0.0373	0.0749	0.1233	0.0044	0.0241	0.0157	0.0082	0.0482
GLS	H_0 : CSR $R^2 = 1$	0.0006	0.0013	0.0025	0.0006	0.0005	0.0007	0.0006	0.0006
	H_0 : CSR $R^2 = 0$	0.6052	0.1049	0.0761	0.9308	0.5250	0.6294	0.9482	0.5783

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the volatility of IPG (fitted with AR(1)-GARCH(1,1) model, innovations: first-difference, factor proposed in Boons and Tamoni, 2015). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_1 = 0$) for the GLS case. The test assets are the 25 FF size and investment value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0 = 3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.13: Spectral Zoo: Volatility of IPG from Boons and Tamoni (2015)

size and operating profitability portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.7263	0.5124	0.4757	0.2380	0.5596	0.3960	0.5676	0.4108
	FM t-statistic	3.7550	2.9965	2.6930	1.1518	2.9822	2.1236	3.3978	2.3424
	MR t-statistic	5.1750	1.7703	1.8043	0.7084	1.6419	1.4130	3.1186	1.5742
	boot p-val	0.0000	0.0955	0.1605	0.7770	0.3420	0.2505	0.0100	0.1480
OLS	$\gamma_{1,j}$	0.0005	0.1082	-0.1265	-0.0433	-0.0056	-0.0075	-0.0022	-0.0077
	FM t-statistic	0.0111	2.7692	-2.5304	-2.7301	-0.8634	-2.8404	-1.8707	-2.4600
	MR t-statistic	0.0069	1.8122	-1.4017	-1.5910	-0.4659	-1.3919	-1.4605	-1.2895
	boot p-val	0.9935	0.6805	0.9170	0.6410	0.9020	0.4055	0.1940	0.3520
GLS	$\gamma_{1,j}$	-0.0111	0.0258	-0.0092	-0.0070	0.0002	-0.0015	-0.0008	-0.0021
	FM t-statistic	-0.3312	1.4920	-0.4074	-0.8405	0.0617	-1.2519	-1.5537	-1.5536
	MR t-statistic	-0.2595	1.1258	-0.2434	-0.5271	0.0374	-0.6033	-0.9184	-0.7078
	boot p-val	0.8020	0.6670	0.9665	0.8855	0.9760	0.5410	0.3375	0.4645
OLS	CSR R^2	0.000%	42.090%	34.719%	47.197%	5.307%	42.615%	36.338%	49.183%
	$se\left(\widehat{R^2}\right)$	0.0004	0.2435	0.3820	0.2837	0.2443	0.2701	0.3133	0.2928
GLS	CSR R^2	0.260%	5.279%	0.394%	1.675%	0.009%	3.716%	5.724%	5.724%
	$se\left(\widehat{R^2}\right)$	0.0177	0.1052	0.0307	0.0646	0.0042	0.1170	0.1256	0.1532
GLS	H_0 : CSR $R^2=1$	0.0921	0.1330	0.1196	0.1338	0.0887	0.1244	0.0630	0.1075
	H_0 : CSR $R^2=0$	0.7942	0.2761	0.8029	0.6061	0.9703	0.5415	0.3993	0.4743

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.8920	0.6463	0.5299	0.9742	0.8016	0.6955	0.6137	0.5525
	MR t-statistic	1.8332	2.6326	2.4308	1.8944	1.7606	3.3060	2.3291	1.4932
	boot p-val	0.1710	0.0090	0.0260	0.0485	0.0835	0.0055	0.0145	0.2350
OLS	$\gamma_{1,n}$	0.0089	-0.0005	-0.0165	0.0370	0.0172	-0.0290	-0.0382	0.1197
	MR t-statistic	0.4532	-0.0111	-0.7566	0.9426	0.4444	-0.6825	-0.8041	1.4155
	boot p-val	0.6305	0.9910	0.5120	0.3410	0.7080	0.5370	0.5320	0.3085
GLS	$\gamma_{1,n}$	0.0041	0.0142	0.0113	-0.0032	-0.0073	-0.0048	-0.0031	0.0374
	MR t-statistic	0.7509	1.1432	1.1434	-0.1853	-0.4804	-0.2952	-0.2775	1.2915
	boot p-val	0.4000	0.3690	0.2555	0.8180	0.5505	0.7815	0.7515	0.2325
OLS	CSR R^2	4.682%	0.003%	7.065%	12.455%	3.073%	6.409%	9.516%	51.839%
	$se\left(\widehat{R^2}\right)$	0.2446	0.0057	0.1591	0.2576	0.1366	0.1928	0.2611	0.4178
GLS	CSR R^2	2.731%	7.015%	4.903%	0.251%	1.185%	0.500%	0.248%	15.608%
	$se\left(\widehat{R^2}\right)$	0.0795	0.1059	0.0830	0.0255	0.0351	0.0361	0.0185	0.2015
GLS	H_0 : CSR $R^2=1$	0.0989	0.1273	0.1074	0.1038	0.1242	0.0809	0.0894	0.3453
	H_0 : CSR $R^2=0$	0.4983	0.1652	0.2585	0.8482	0.6171	0.7718	0.7804	0.1199

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the volatility of IPG (fitted with AR(1)-GARCH(1,1) model, innovations: first-difference, factor proposed in Boons and Tamoni, 2015). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0 : R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0 : \gamma_1=0$) for the GLS case. The test assets are the 25 FF size and operating profitability value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.14: Spectral Zoo: Financial Uncertainty Index

size and book-to-market portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j = 1$ $\left[\frac{1}{2}, \frac{1}{4}\right]$	2 $\left[\frac{1}{4}, \frac{1}{8}\right]$	3 $\left[\frac{1}{8}, \frac{1}{16}\right]$	4 $\left[\frac{1}{16}, \frac{1}{32}\right]$	5 $\left[\frac{1}{32}, \frac{1}{64}\right]$	6 $\left[\frac{1}{64}, \frac{1}{128}\right]$	> 6 $\left[\frac{1}{128}, 0\right]$	> 5 $\left[\frac{1}{64}, 0\right]$
OLS	$\gamma_{0,j}$	0.5665	0.5509	0.5129	0.6161	0.5171	0.3267	0.7296	0.4500
	FM t-statistic	3.1006	1.9299	1.9255	2.8673	2.5184	1.6486	3.6967	2.2679
	MR t-statistic	1.7358	1.4854	1.4043	1.2310	2.2416	1.1946	2.5727	1.7950
	boot p-val	0.1055	0.2265	0.2195	0.3065	0.0580	0.3850	0.0045	0.1285
OLS	$\gamma_{1,j}$	-0.3781	-0.2070	-0.3056	-0.1407	-0.2515	-0.4677	-0.3046	-0.5781
	FM t-statistic	-0.8346	-0.5596	-0.7217	-0.4874	-0.9890	-3.4664	-3.4355	-3.5948
	MR t-statistic	-0.5671	-0.4660	-0.5674	-0.2327	-0.8036	-1.5801	-1.8249	-1.8037
	boot p-val	0.6580	0.7035	0.6320	0.8245	0.4620	0.2580	0.1940	0.1470
GLS	$\gamma_{1,j}$	0.2984	0.2856	-0.0089	0.4060	0.1275	-0.1323	-0.1129	-0.2249
	FM t-statistic	1.3410	1.3858	-0.0459	2.9476	0.9573	-1.7674	-2.1876	-2.2336
	MR t-statistic	0.6927	0.8400	-0.0316	2.2018	0.9418	-0.7277	-1.1545	-0.8937
	boot p-val	0.4780	0.3990	0.9785	0.0510	0.1720	0.4580	0.3450	0.4105
OLS	CSR R^2	5.077%	3.028%	4.821%	1.024%	8.023%	49.630%	52.152%	55.267%
	$se\left(\widehat{R^2}\right)$	0.1860	0.1278	0.1624	0.0870	0.1768	0.3037	0.2550	0.2696
GLS	CSR R^2	2.319%	2.476%	0.003%	11.204%	1.182%	4.028%	6.171%	6.433%
	$se\left(\widehat{R^2}\right)$	0.0674	0.0533	0.0016	0.0985	0.0266	0.1071	0.1023	0.1231
GLS	H_0 : CSR $R^2 = 1$	0.0001	0.0001	0.0017	0.0039	0.0024	0.0115	0.0119	0.0194
	H_0 : CSR $R^2 = 0$	0.5041	0.4018	0.9748	0.0872	0.4181	0.2954	0.2440	0.3107

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n = 0$ $\left[0, \frac{1}{16}\right]$	1 $\left[\frac{1}{16}, \frac{1}{8}\right]$	2 $\left[\frac{1}{8}, \frac{3}{16}\right]$	3 $\left[\frac{3}{16}, \frac{1}{4}\right]$	4 $\left[\frac{1}{4}, \frac{5}{16}\right]$	5 $\left[\frac{5}{16}, \frac{3}{8}\right]$	6 $\left[\frac{3}{8}, \frac{7}{16}\right]$	7 $\left[\frac{7}{16}, \frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.4529	0.6379	0.7805	0.7278	0.7053	0.7333	0.8567	0.7223
	MR t-statistic	1.1151	1.7191	2.7267	1.4864	1.7609	3.2734	3.0085	2.4449
	boot p-val	0.3675	0.1270	0.0070	0.2065	0.1215	0.0045	0.0070	0.0050
OLS	$\gamma_{1,n}$	-0.3386	-0.0490	0.1171	0.0430	0.0181	0.1742	0.5553	-0.3731
	MR t-statistic	-0.4739	-0.0993	0.3245	0.0932	0.0609	0.2847	0.8430	-0.6368
	boot p-val	0.6430	0.9245	0.7170	0.9270	0.9570	0.8070	0.3980	0.5735
GLS	$\gamma_{1,n}$	0.3238	0.0540	0.4266	0.1422	-0.0149	-0.0649	0.2649	0.4023
	MR t-statistic	1.3750	0.1778	1.2028	0.6187	-0.0758	-0.1461	1.1282	1.4267
	boot p-val	0.1115	0.8200	0.2735	0.4890	0.9315	0.9020	0.4360	0.2450
OLS	CSR R^2	3.534%	0.119%	0.957%	0.140%	0.051%	1.552%	17.746%	5.745%
	$se\left(\widehat{R^2}\right)$	0.1334	0.0244	0.0568	0.0298	0.0165	0.1054	0.3858	0.1814
GLS	CSR R^2	3.410%	0.096%	5.738%	1.202%	0.021%	0.211%	6.053%	11.620%
	$se\left(\widehat{R^2}\right)$	0.0529	0.0092	0.0772	0.0398	0.0061	0.0287	0.0992	0.1738
GLS	H_0 : CSR $R^2 = 1$	0.0021	0.0021	0.0027	0.0013	0.0021	0.0013	0.0041	0.0132
	H_0 : CSR $R^2 = 0$	0.2737	0.8555	0.2241	0.5529	0.9404	0.8824	0.2584	0.1834

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) financial uncertainty index (innovations: first-difference, $h = 1$, the factor is priced at $j = 6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_1 = 0$) for the GLS case. The test assets are the 25 FF size and book-to-market value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0 = 3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.15: Spectral Zoo: Financial Uncertainty Index

size and investment portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j = 1$ $\left[\frac{1}{2}, \frac{1}{4}\right]$	2 $\left[\frac{1}{4}, \frac{1}{8}\right]$	3 $\left[\frac{1}{8}, \frac{1}{16}\right]$	4 $\left[\frac{1}{16}, \frac{1}{32}\right]$	5 $\left[\frac{1}{32}, \frac{1}{64}\right]$	6 $\left[\frac{1}{64}, \frac{1}{128}\right]$	>6 $\left[\frac{1}{128}, 0\right]$	>5 $\left[\frac{1}{64}, 0\right]$
OLS	$\gamma_{0,j}$	0.4189	0.5532	0.5528	0.6125	0.5489	0.3535	0.7575	0.5013
	FM t-statistic	2.0818	2.2164	2.4963	2.4539	2.6581	1.8689	3.8603	2.6962
	MR t-statistic	1.5525	1.7653	1.8244	1.5766	2.2593	1.1311	3.1801	1.8712
	boot p-val	0.1955	0.1935	0.1225	0.2415	0.0710	0.3635	0.0015	0.1065
OLS	$\gamma_{1,j}$	-0.7174	-0.2205	-0.2691	-0.1617	-0.2364	-0.4634	-0.2537	-0.5264
	FM t-statistic	-1.4151	-0.6330	-0.6978	-0.4388	-0.8397	-2.9020	-3.3309	-3.1124
	MR t-statistic	-0.9164	-0.5317	-0.5327	-0.3102	-0.7347	-1.4599	-2.1618	-1.7126
	boot p-val	0.4565	0.6650	0.6570	0.8245	0.4915	0.2495	0.0870	0.1310
GLS	$\gamma_{1,j}$	-0.5087	0.2019	-0.3076	0.3705	-0.1164	-0.2252	-0.1281	-0.3011
	FM t-statistic	-1.8606	1.0011	-1.4099	1.9352	-0.6913	-2.6598	-2.7137	-2.9929
	MR t-statistic	-0.8361	0.5176	-0.6707	1.0763	-0.4055	-1.3101	-1.4564	-1.5491
	boot p-val	0.5720	0.5560	0.8195	0.2615	0.6830	0.2980	0.1855	0.1920
OLS	CSR R^2	19.308%	4.110%	4.750%	1.751%	8.061%	56.809%	51.636%	58.780%
	$se\left(\widehat{R^2}\right)$	0.3151	0.1477	0.1725	0.1132	0.2040	0.3200	0.2358	0.2670
GLS	CSR R^2	3.787%	1.096%	2.174%	4.097%	0.523%	7.739%	8.055%	9.798%
	$se\left(\widehat{R^2}\right)$	0.0989	0.0427	0.0656	0.0736	0.0245	0.1162	0.1042	0.1191
GLS	H_0 : CSR $R^2 = 1$	0.0032	0.0003	0.0024	0.0009	0.0013	0.0084	0.0057	0.0112
	H_0 : CSR $R^2 = 0$	0.4394	0.5942	0.4952	0.3034	0.6776	0.1930	0.1435	0.1189

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n = 0$ $\left[0, \frac{1}{16}\right]$	1 $\left[\frac{1}{16}, \frac{1}{8}\right]$	2 $\left[\frac{1}{8}, \frac{3}{16}\right]$	3 $\left[\frac{3}{16}, \frac{1}{4}\right]$	4 $\left[\frac{1}{4}, \frac{5}{16}\right]$	5 $\left[\frac{5}{16}, \frac{3}{8}\right]$	6 $\left[\frac{3}{8}, \frac{7}{16}\right]$	7 $\left[\frac{7}{16}, \frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.4850	0.6802	0.8341	0.5046	0.5553	0.7281	0.6578	0.7863
	MR t-statistic	1.6685	2.3273	3.1195	1.2318	1.1940	3.0797	3.0444	1.9563
	boot p-val	0.1720	0.0445	0.0050	0.3170	0.3190	0.0105	0.0100	0.0170
OLS	$\gamma_{1,n}$	-0.3077	-0.0038	0.1798	-0.1589	-0.0880	0.1414	-0.0838	-0.8629
	MR t-statistic	-0.5087	-0.0088	0.4921	-0.4003	-0.2633	0.2706	-0.1336	-1.2457
	boot p-val	0.6150	0.9935	0.6005	0.7205	0.8125	0.7950	0.8840	0.5070
GLS	$\gamma_{1,n}$	0.1823	0.1324	0.3649	-0.0269	-0.0367	0.0041	-0.0383	-0.1936
	MR t-statistic	0.4344	0.3849	1.1878	-0.0621	-0.1032	0.0189	-0.1637	-0.7407
	boot p-val	0.6535	0.6110	0.1845	0.9515	0.9115	0.9875	0.8690	0.5125
OLS	CSR R^2	3.689%	0.001%	2.729%	1.986%	0.986%	1.308%	0.356%	32.593%
	$se\left(\widehat{R^2}\right)$	0.1247	0.0021	0.1064	0.1030	0.0766	0.0946	0.0537	0.2440
GLS	CSR R^2	0.542%	0.410%	5.461%	0.027%	0.080%	0.002%	0.101%	1.572%
	$se\left(\widehat{R^2}\right)$	0.0251	0.0212	0.0902	0.0085	0.0155	0.0017	0.0138	0.0454
GLS	H_0 : CSR $R^2 = 1$	0.0006	0.0004	0.0004	0.0008	0.0008	0.0006	0.0007	0.0007
	H_0 : CSR $R^2 = 0$	0.6723	0.7038	0.2426	0.9506	0.9183	0.9849	0.8701	0.4739

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) financial uncertainty index (innovations: first-difference, $h = 1$, the factor is priced at $j = 6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \gamma_1 = 0$) for the GLS case. The test assets are the 25 FF size and investment value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0 = 3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.16: Spectral Zoo: Financial Uncertainty Index

size and operating profitability portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.3250	0.0716	0.2555	0.1232	0.3894	0.3761	0.7224	0.4873
	FM t-statistic	1.6926	0.2875	1.1322	0.4892	2.0031	2.0539	3.6539	2.7031
	MR t-statistic	1.1212	0.2003	0.8409	0.2668	1.3947	1.2820	2.8671	1.9934
	boot p-val	0.3915	0.8625	0.5325	0.8405	0.3085	0.3280	0.0025	0.0825
OLS	$\gamma_{1,j}$	-0.8213	-0.6853	-0.6072	-0.6485	-0.3738	-0.4061	-0.2296	-0.4931
	FM t-statistic	-1.7583	-2.1104	-1.6310	-1.9179	-1.5390	-2.1026	-2.8445	-2.5176
	MR t-statistic	-1.1378	-1.4668	-1.2360	-1.0795	-1.1959	-1.1627	-1.8928	-1.3605
	boot p-val	0.4310	0.1550	0.3985	0.4000	0.3065	0.3555	0.1380	0.2390
GLS	$\gamma_{1,j}$	0.2006	-0.0225	0.0150	-0.0820	-0.1364	-0.0353	0.0034	-0.0211
	FM t-statistic	0.8760	-0.1193	0.0723	-0.5159	-1.1364	-0.3436	0.0680	-0.1825
	MR t-statistic	0.5202	-0.0766	0.0519	-0.2640	-0.6450	-0.2122	0.0365	-0.1175
	boot p-val	0.6470	0.9425	0.9595	0.8335	0.4790	0.7920	0.9670	0.8775
OLS	CSR R^2	29.041%	43.376%	26.688%	31.085%	23.838%	34.702%	52.653%	47.540%
	$se\left(\widehat{R^2}\right)$	0.3234	0.3298	0.3508	0.3384	0.3013	0.3693	0.3091	0.2967
GLS	CSR R^2	1.820%	0.034%	0.012%	0.631%	3.062%	0.280%	0.011%	0.079%
	$se\left(\widehat{R^2}\right)$	0.0683	0.0086	0.0039	0.0467	0.0693	0.0258	0.0059	0.0130
GLS	H_0 : CSR $R^2=1$	0.0949	0.0686	0.0526	0.1164	0.1230	0.0965	0.0631	0.0740
	H_0 : CSR $R^2=0$	0.5979	0.9390	0.9585	0.7877	0.4968	0.8282	0.9708	0.9059

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.2269	0.3605	0.1558	0.1260	0.0795	0.6010	0.6422	0.6037
	MR t-statistic	0.5899	1.0574	0.5164	0.3338	0.1674	1.9396	2.4177	1.9889
	boot p-val	0.6365	0.3875	0.5840	0.7325	0.8735	0.0340	0.0340	0.0375
OLS	$\gamma_{1,n}$	-0.6481	-0.3457	-0.5768	-0.4541	-0.3851	-0.1419	-0.0207	0.3592
	MR t-statistic	-0.9919	-0.8048	-1.2020	-1.1991	-1.1203	-0.1845	-0.0265	0.2843
	boot p-val	0.4000	0.4810	0.2120	0.2760	0.3160	0.8445	0.9790	0.7560
GLS	$\gamma_{1,n}$	-0.1368	0.1238	0.0085	0.0892	0.0609	0.0283	0.0422	0.3840
	MR t-statistic	-0.3549	0.4133	0.0375	0.4458	0.3491	0.1026	0.2017	1.3720
	boot p-val	0.7440	0.6280	0.9655	0.6695	0.7660	0.9270	0.8180	0.4110
OLS	CSR R^2	22.022%	9.633%	29.510%	21.666%	25.922%	1.173%	0.022%	6.444%
	$se\left(\widehat{R^2}\right)$	0.3527	0.2280	0.3722	0.3113	0.3351	0.1306	0.0163	0.4197
GLS	CSR R^2	1.059%	0.879%	0.008%	1.213%	0.836%	0.105%	0.211%	16.131%
	$se\left(\widehat{R^2}\right)$	0.0532	0.0369	0.0042	0.0535	0.0479	0.0201	0.0210	0.2454
GLS	H_0 : CSR $R^2=1$	0.1024	0.0836	0.0709	0.1103	0.1104	0.0550	0.0865	0.3554
	H_0 : CSR $R^2=0$	0.7096	0.6842	0.9701	0.6649	0.7278	0.9182	0.8428	0.1828

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) financial uncertainty index (innovations: first-difference, $h=1$, the factor is priced at $j=6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0 : R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0 : \gamma_1=0$) for the GLS case. The test assets are the 25 FF size and operating profitability value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.17: Spectral Zoo: Real Uncertainty Index

size and book-to-market portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.5194	0.2800	0.1033	0.3853	0.2674	0.5187	0.7796	0.6164
	FM t-statistic	2.9630	1.2307	0.5111	2.2015	1.3302	2.5903	4.0054	3.0331
	MR t-statistic	1.1785	0.5861	0.3350	0.7302	0.8167	1.8122	3.3664	2.3776
	boot p-val	0.4660	0.6795	0.9635	0.7195	0.8200	0.0800	0.0010	0.0305
OLS	$\gamma_{1,j}$	-0.3120	-0.6505	-1.6181	-0.5053	-0.3318	-0.1586	-0.1508	-0.2027
	FM t-statistic	-1.0520	-1.6893	-3.7387	-2.1774	-2.6261	-3.5162	-2.6046	-3.3500
	MR t-statistic	-0.5264	-0.9528	-1.5428	-0.7243	-1.2163	-1.7119	-1.3495	-1.7726
	boot p-val	0.8200	0.5390	0.8225	0.7270	0.6790	0.1295	0.2845	0.0880
GLS	$\gamma_{1,j}$	-0.0563	0.0024	-0.7042	0.0452	-0.0747	-0.0759	-0.0986	-0.1350
	FM t-statistic	-0.4448	0.0138	-3.2558	0.3333	-1.2221	-2.4069	-3.3229	-3.2730
	MR t-statistic	-0.2581	0.0081	-1.8061	0.1209	-0.4838	-1.1431	-1.5289	-1.5593
	boot p-val	0.7675	0.9905	0.9240	0.9280	0.7430	0.3135	0.3735	0.1875
OLS	CSR R^2	4.641%	13.377%	55.977%	10.294%	29.558%	61.141%	46.117%	61.935%
	$se\left(\widehat{R^2}\right)$	0.2056	0.2876	0.3560	0.3559	0.3838	0.2452	0.3144	0.3190
GLS	CSR R^2	0.255%	0.000%	13.669%	0.143%	1.926%	7.470%	14.239%	13.814%
	$se\left(\widehat{R^2}\right)$	0.0194	0.0006	0.1557	0.0234	0.0787	0.1245	0.1497	0.1546
GLS	H_0 : CSR $R^2=1$	0.0031	0.0014	0.1109	0.0022	0.0069	0.0110	0.0183	0.0150
	H_0 : CSR $R^2=0$	0.7917	0.9935	0.0861	0.9024	0.6196	0.2150	0.0694	0.0519

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.5585	0.7132	0.5318	0.7824	0.6603	0.9349	0.8735	0.8634
	MR t-statistic	0.8061	2.4529	1.3470	1.5103	1.7986	2.9731	4.3201	2.9763
	boot p-val	0.4235	0.0160	0.1790	0.1535	0.0305	0.0095	0.0025	0.0020
OLS	$\gamma_{1,n}$	-0.1260	0.1288	-0.1853	0.1128	0.3467	0.2897	0.2359	0.1939
	MR t-statistic	-0.1595	0.1494	-0.3764	0.2071	0.6531	0.8734	0.9134	0.6213
	boot p-val	0.8655	0.8705	0.7020	0.8230	0.6775	0.6325	0.5505	0.5860
GLS	$\gamma_{1,n}$	0.1354	0.1223	0.1943	-0.0483	-0.0683	0.0632	0.0586	0.0439
	MR t-statistic	0.6621	0.5109	1.0587	-0.4155	-0.4248	0.3216	0.5515	0.2606
	boot p-val	0.4095	0.5805	0.2315	0.7330	0.6280	0.8410	0.6455	0.8625
OLS	CSR R^2	0.777%	0.473%	2.393%	1.049%	6.278%	10.369%	5.886%	9.281%
	$se\left(\widehat{R^2}\right)$	0.0939	0.0619	0.1178	0.1054	0.2334	0.2121	0.1382	0.2011
GLS	CSR R^2	2.119%	0.874%	3.519%	0.632%	0.956%	0.658%	1.232%	0.481%
	$se\left(\widehat{R^2}\right)$	0.0678	0.0339	0.0521	0.0303	0.0436	0.0366	0.0350	0.0365
GLS	H_0 : CSR $R^2=1$	0.0015	0.0019	0.0026	0.0011	0.0013	0.0034	0.0027	0.0027
	H_0 : CSR $R^2=0$	0.5460	0.6037	0.2641	0.6765	0.6602	0.7446	0.5687	0.7925

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) real uncertainty index (innovations: first-difference, $h=1$, the factor is priced at $j=6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0: R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0:\gamma_1=0$) for the GLS case. The test assets are the 25 FF size and book-to-market value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.18: Spectral Zoo: Real Uncertainty Index

size and investment portfolios

Panel A		Beta decomposition based on MODWT MRA, Indirect							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.7252	0.6646	0.5160	0.4258	0.2973	0.5548	0.8073	0.6352
	FM t-statistic	3.7172	3.3004	2.7384	2.0676	1.2809	3.0100	4.2185	3.2503
	MR t-statistic	1.3430	1.8323	1.4051	0.7865	0.7495	2.2307	3.4895	2.5208
	boot p-val	0.2135	0.1010	0.3770	0.7550	0.8045	0.0360	0.0005	0.0195
OLS	$\gamma_{1,j}$	-0.0474	-0.1340	-0.6082	-0.4735	-0.3206	-0.1525	-0.1565	-0.2213
	FM t-statistic	-0.1316	-0.3527	-1.2826	-1.2796	-1.6377	-2.7994	-3.3341	-3.5780
	MR t-statistic	-0.0581	-0.2437	-0.7367	-0.5968	-0.9511	-1.7060	-1.7997	-2.2145
	boot p-val	0.9765	0.8350	0.8270	0.8060	0.7550	0.1600	0.2615	0.0650
GLS	$\gamma_{1,j}$	0.1306	0.1262	-0.0694	-0.3106	-0.4216	-0.0345	-0.1153	-0.1155
	FM t-statistic	0.9982	0.8903	-0.3921	-2.0377	-4.8347	-1.0788	-3.6845	-2.6341
	MR t-statistic	0.5273	0.5667	-0.1890	-0.6939	-2.0275	-0.5945	-1.7190	-1.3201
	boot p-val	0.5960	0.6290	0.9545	0.8570	0.7185	0.5850	0.3350	0.3025
OLS	CSR R^2	0.103%	0.900%	10.147%	10.734%	26.228%	51.089%	44.977%	60.498%
	$se\left(\widehat{R^2}\right)$	0.0364	0.0686	0.2685	0.3904	0.4962	0.3013	0.3530	0.2176
GLS	CSR R^2	1.090%	0.867%	0.168%	4.542%	25.568%	1.273%	14.850%	7.589%
	$se\left(\widehat{R^2}\right)$	0.0394	0.0293	0.0182	0.1366	0.2198	0.0445	0.1569	0.1161
GLS	H_0 : CSR $R^2=1$	0.0019	0.0004	0.0014	0.0057	0.3601	0.0014	0.0082	0.0017
	H_0 : CSR $R^2=0$	0.5878	0.5635	0.8525	0.4946	0.0144	0.5488	0.0549	0.1568

Panel B		Beta decomposition based on MODWPT MRA, Indirect							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.8004	0.8675	0.6928	1.0292	0.6681	0.9353	0.9813	0.9108
	MR t-statistic	1.2992	3.3481	2.9145	2.3125	1.9648	3.1343	2.3326	3.2749
	boot p-val	0.1340	0.0005	0.0060	0.0940	0.0310	0.0045	0.0165	0.0025
OLS	$\gamma_{1,n}$	0.1219	0.6763	0.0123	0.3817	0.3495	0.2776	0.3497	0.2363
	MR t-statistic	0.1663	0.8963	0.0475	0.7845	1.1225	0.9849	1.1061	0.8077
	boot p-val	0.8330	0.4025	0.9645	0.5535	0.2840	0.5930	0.3710	0.4975
GLS	$\gamma_{1,n}$	0.1926	0.4746	-0.0365	0.0702	0.1387	0.0301	0.0731	0.1191
	MR t-statistic	0.5334	1.8046	-0.1766	0.5441	1.0669	0.2860	0.7463	0.9043
	boot p-val	0.5965	0.1600	0.8810	0.5100	0.2410	0.7250	0.4415	0.4385
OLS	CSR R^2	0.694%	14.974%	0.016%	12.150%	11.487%	10.822%	14.944%	14.638%
	$se\left(\widehat{R^2}\right)$	0.0849	0.2652	0.0073	0.3249	0.1920	0.2129	0.2754	0.3088
GLS	CSR R^2	1.583%	10.564%	0.135%	0.982%	3.605%	0.213%	2.383%	4.089%
	$se\left(\widehat{R^2}\right)$	0.0582	0.1051	0.0157	0.0303	0.0489	0.0140	0.0641	0.0912
GLS	H_0 : CSR $R^2=1$	0.0009	0.0064	0.0001	0.0008	0.0020	0.0007	0.0007	0.0025
	H_0 : CSR $R^2=0$	0.5905	0.0642	0.8612	0.5705	0.3024	0.7724	0.4573	0.3720

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) real uncertainty index (innovations: first-difference, $h=1$, the factor is priced at $j=6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0 : R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0 : \gamma_1=0$) for the GLS case. The test assets are the 25 FF size and investment value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.19: Spectral Zoo: Real Uncertainty Index

size and operating profitability portfolios

Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**							
Frequency in cycles per period		$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$	>5 $\left[\frac{1}{64},0\right]$
OLS	$\gamma_{0,j}$	0.2978	0.1723	0.3208	0.2242	0.3800	0.5407	0.7671	0.6221
	FM t-statistic	1.6301	0.8223	1.6821	1.1208	1.8926	2.8973	3.9019	3.2189
	MR t-statistic	0.9351	0.5236	1.0991	0.6883	1.1244	2.2328	3.7078	2.7000
	boot p-val	0.6790	0.6135	0.4110	0.6245	0.4245	0.0325	0.0015	0.0145
OLS	$\gamma_{1,j}$	-0.5826	-0.7660	-0.9985	-0.6820	-0.2381	-0.1402	-0.1210	-0.1927
	FM t-statistic	-1.8320	-2.3550	-2.9513	-2.7390	-1.6155	-2.8007	-2.3545	-2.9560
	MR t-statistic	-1.1989	-1.5634	-1.7005	-1.5936	-0.9711	-1.4714	-1.3415	-1.6678
	boot p-val	0.6245	0.1985	0.4130	0.3390	0.4950	0.1835	0.5060	0.1225
GLS	$\gamma_{1,j}$	0.0054	0.1634	-0.1504	-0.0485	-0.0479	-0.0520	-0.0042	-0.0615
	FM t-statistic	0.0362	1.1648	-0.8185	-0.3920	-0.7577	-1.7950	-0.1252	-1.4135
	MR t-statistic	0.0253	0.7630	-0.5618	-0.2552	-0.4145	-0.9865	-0.0739	-0.8096
	boot p-val	0.9750	0.5630	0.7950	0.8700	0.8190	0.2845	0.9410	0.3845
OLS	CSR R^2	19.890%	39.008%	58.869%	45.945%	21.202%	60.627%	24.073%	55.868%
	$se\left(\widehat{R^2}\right)$	0.2882	0.1895	0.1707	0.3216	0.3997	0.2564	0.3025	0.2307
GLS	CSR R^2	0.003%	3.217%	1.589%	0.364%	1.361%	7.640%	0.037%	4.738%
	$se\left(\widehat{R^2}\right)$	0.0018	0.0854	0.0573	0.0283	0.0628	0.1327	0.0101	0.0998
GLS	H_0 : CSR $R^2=1$	0.0908	0.1031	0.0929	0.1155	0.1165	0.1257	0.0869	0.0909
	H_0 : CSR $R^2=0$	0.9797	0.4371	0.5782	0.7992	0.6620	0.3024	0.9412	0.3933

Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**							
Frequency in cycles per period		$n=0$ $\left[0,\frac{1}{16}\right]$	1 $\left[\frac{1}{16},\frac{1}{8}\right]$	2 $\left[\frac{1}{8},\frac{3}{16}\right]$	3 $\left[\frac{3}{16},\frac{1}{4}\right]$	4 $\left[\frac{1}{4},\frac{5}{16}\right]$	5 $\left[\frac{5}{16},\frac{3}{8}\right]$	6 $\left[\frac{3}{8},\frac{7}{16}\right]$	7 $\left[\frac{7}{16},\frac{1}{2}\right]$
OLS	$\gamma_{0,n}$	0.2152	0.4994	0.2186	0.9192	0.6384	0.7114	0.8576	0.6107
	MR t-statistic	0.3592	1.7778	0.7583	1.1920	1.3484	2.2959	2.3893	1.7892
	boot p-val	0.7500	0.0520	0.4670	0.3770	0.0760	0.0170	0.0095	0.0795
OLS	$\gamma_{1,n}$	-0.4538	-0.5645	-0.5534	0.2810	0.5863	0.0697	0.2577	-0.0393
	MR t-statistic	-0.6245	-1.4308	-1.6444	0.3377	1.5520	0.2182	0.6361	-0.1222
	boot p-val	0.5520	0.2545	0.1660	0.8200	0.3325	0.8180	0.5550	0.9110
GLS	$\gamma_{1,n}$	0.0244	0.0707	-0.0827	0.0993	0.1773	0.0110	0.0136	-0.1482
	MR t-statistic	0.1037	0.3345	-0.4796	1.0087	1.7091	0.0998	0.1525	-1.0923
	boot p-val	0.9060	0.6870	0.7120	0.4480	0.2045	0.9380	0.8725	0.4865
OLS	CSR R^2	15.598%	21.010%	49.439%	5.660%	36.159%	0.967%	10.874%	0.337%
	$se\left(\widehat{R^2}\right)$	0.4624	0.2895	0.2589	0.3401	0.5131	0.0920	0.2961	0.0562
GLS	CSR R^2	0.095%	0.687%	1.691%	4.116%	13.922%	0.058%	0.123%	9.789%
	$se\left(\widehat{R^2}\right)$	0.0166	0.0337	0.0719	0.0823	0.1494	0.0111	0.0132	0.0995
GLS	H_0 : CSR $R^2=1$	0.0926	0.0795	0.0892	0.1665	0.3582	0.0937	0.0700	0.1486
	H_0 : CSR $R^2=0$	0.9187	0.7321	0.6398	0.2948	0.1078	0.9208	0.8790	0.2005

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the Jurado et al. (2015) real uncertainty index (innovations: first-difference, $h=1$, the factor is priced at $j=6$ in Xyngis, 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing H_0 : $R^2=1$) and H_0 : CSR $R^2=0$ (imposing H_0 : $\gamma_1=0$) for the GLS case. The test assets are the 25 FF size and operating profitability value-weighted portfolios and the data are monthly from July 1963 through December 2020. In Panel A the beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and in Panel B based on the MODWPT MRA indirect method with $J_0=3$ (using only boundary independent coefficients). In both cases the filter is LA(8).

Table IA.20: Spectral Zoo: Consumption Growth from Bandi and Tamoni (2013) and Kang et al. (2017)

size and book-to-market portfolios: 1949:Q1 - 2018:Q4

	Frequency in cycles per period	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	>4 $\left[\frac{1}{32},0\right]$	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	>4 $\left[\frac{1}{32},0\right]$
		NIPA nondurables and services					**Parker and Julliard - consumption growth**				
OLS	$\gamma_{0,j}$	2.6106	1.7342	1.5764	0.8827	1.9582	2.6431	2.6347	1.7461	1.3968	1.9915
	MR t-statistic	4.8515	3.1060	2.4381	0.5761	1.7376	6.8023	4.0474	3.2526	1.7436	1.7936
	boot p-val	0.0040	0.0580	0.0525	0.5225	0.1690	0.0020	0.0075	0.0415	0.2200	0.1730
OLS	$\gamma_{1,j}$	-0.0292	0.1500	0.1596	0.1278	0.0490	-0.2527	0.2457	0.1919	0.5027	0.2745
	MR t-statistic	-0.1494	1.7596	1.2714	0.9291	0.7756	-1.0659	2.3972	2.2468	1.6158	0.7595
	boot p-val	0.8830	0.0650	0.2295	0.2815	0.4715	0.4560	0.0835	0.0445	0.1950	0.5910
GLS	$\gamma_{1,j}$	0.0967	0.1309	0.0075	0.0777	-0.0064	-0.0521	0.1193	0.0534	0.0211	0.0464
	MR t-statistic	0.5075	1.4568	0.0735	1.0569	-0.2075	-0.2670	1.8349	1.1308	0.1475	0.3975
	boot p-val	0.6405	0.1730	0.9380	0.3745	0.8450	0.7930	0.3455	0.2375	0.8835	0.7005
OLS	CSR R^2	0.312%	29.128%	13.360%	21.643%	12.595%	4.707%	59.341%	47.402%	44.924%	25.767%
	$se\left(\widehat{R^2}\right)$	0.0421	0.2904	0.2260	0.4970	0.3248	0.1745	0.2454	0.2549	0.3420	0.5990
GLS	CSR R^2	1.279%	9.425%	0.024%	7.774%	0.229%	0.332%	10.587%	2.260%	0.128%	0.898%
	$se\left(\widehat{R^2}\right)$	0.0531	0.0919	0.0065	0.1159	0.0214	0.0222	0.1082	0.0374	0.0139	0.0429
GLS	H_0 : CSR $R^2=1$	0.0045	0.0084	0.0064	0.0350	0.0052	0.0011	0.0186	0.0075	0.0084	0.0108
	H_0 : CSR $R^2=0$	0.6227	0.1392	0.9414	0.2653	0.8388	0.7925	0.0809	0.2988	0.8829	0.6844
		Kroencke - unfiltered NIPA nondurables and services					**Kroencke - unfiltered NIPA nondurables**				
OLS	$\gamma_{0,j}$	2.6897	1.8487	1.7024	1.3847	1.9115	2.7253	1.6239	1.2918	0.7360	1.5255
	MR t-statistic	9.7699	3.7026	2.7962	1.1199	1.6443	6.0899	2.8795	1.5857	0.6668	1.5462
	boot p-val	0.0005	0.0520	0.0300	0.1325	0.1650	0.0020	0.0380	0.2190	0.4165	0.2310
OLS	$\gamma_{1,j}$	-0.1465	0.4669	0.3091	0.1390	0.0845	-0.2657	0.5868	0.5775	0.2475	0.1347
	MR t-statistic	-0.9645	2.6089	1.1913	0.7799	0.8636	-0.5620	2.2836	1.6673	1.3019	1.6390
	boot p-val	0.3970	0.0435	0.2750	0.2440	0.4790	0.6335	0.0665	0.1625	0.1260	0.2505
GLS	$\gamma_{1,j}$	0.0155	0.3558	0.0755	0.0774	0.0094	0.3393	0.1553	-0.0567	0.1085	0.0048
	MR t-statistic	0.0308	1.9576	0.3575	0.7624	0.2262	0.5957	0.4823	-0.2501	0.9419	0.1130
	boot p-val	0.9695	0.1420	0.7210	0.4840	0.8245	0.6030	0.6095	0.8110	0.3740	0.9050
OLS	CSR R^2	1.850%	46.557%	12.246%	13.477%	23.305%	2.985%	51.101%	25.360%	34.270%	44.189%
	$se\left(\widehat{R^2}\right)$	0.0981	0.2900	0.2253	0.3300	0.4775	0.1278	0.3391	0.2303	0.4524	0.3505
GLS	CSR R^2	0.006%	13.290%	0.530%	2.834%	0.300%	1.965%	1.450%	0.216%	4.704%	0.054%
	$se\left(\widehat{R^2}\right)$	0.0037	0.1170	0.0305	0.0625	0.0247	0.0640	0.0705	0.0193	0.0942	0.0094
GLS	H_0 : CSR $R^2=1$	0.0016	0.0131	0.0057	0.0116	0.0078	0.0097	0.0052	0.0056	0.0194	0.0063
	H_0 : CSR $R^2=0$	0.9753	0.1139	0.7107	0.4359	0.8172	0.5524	0.6634	0.8037	0.3507	0.9088

Notes: This table reports estimates for the zero-beta excess return (OLS) and the price of frequency-specific beta risk (OLS and GLS) for consumption growth (innovations: first-difference, the factor is priced at $j=4$ in Bandi and Tamoni, 2013 and at $j>4$ in the IA of Kang et al., 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) t-statistics using a HAC variance-covariance matrix. The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0 : R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0 : \gamma_{1,j}=0$) for the GLS case. The beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and the filter is LA(8). The test assets are the 25 FF size and book-to-market value-weighted portfolios and the data are quarterly from 1949:Q1 to 2018:Q4. We use four different measures of consumption growth from Kroencke (2017), namely: NIPA nondurables and services, three-year NIPA consumption (Parker and Julliard, 2005), unfiltered NIPA nondurables and services and unfiltered NIPA nondurables.

Table IA.21: Spectral Zoo: Consumption Growth from Bandi and Tamoni (2013) and Kang et al. (2017)

size and investment portfolios: 1963:Q3 - 2018:Q4

	Frequency in cycles per period	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	>4 $\left[\frac{1}{32},0\right]$	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	>4 $\left[\frac{1}{32},0\right]$
		NIPA nondurables and services					**Parker and Julliard - consumption growth**				
OLS	$\gamma_{0,j}$	2.0150	1.5302	1.5365	1.2386	2.2023	2.3126	2.4911	1.6065	1.5386	1.2365
	MR t-statistic	4.1248	2.6444	1.3694	0.5403	0.5976	8.2704	2.8463	2.2643	1.6657	0.8760
	boot p-val	0.0365	0.1045	0.2635	0.6115	0.6105	0.0005	0.0210	0.1340	0.2435	0.5345
OLS	$\gamma_{1,j}$	0.0707	0.1032	0.1140	0.1048	0.0074	0.1506	0.2484	0.2343	0.3924	0.3591
	MR t-statistic	0.5786	1.5684	0.5782	0.4289	0.0247	0.7313	1.8065	1.9294	1.2992	0.9765
	boot p-val	0.4600	0.1355	0.5830	0.6755	0.9845	0.4710	0.1900	0.1215	0.2975	0.5990
GLS	$\gamma_{1,j}$	-0.1766	0.1210	0.1773	0.0103	0.0015	-0.0835	0.2330	0.1468	-0.1207	0.2210
	MR t-statistic	-1.7454	2.1753	1.1173	0.1704	0.0231	-0.9886	2.7413	2.6458	-1.0775	2.1912
	boot p-val	0.3040	0.0765	0.6075	0.8690	0.9840	0.3940	0.3405	0.1420	0.2305	0.2495
OLS	CSR R^2	5.770%	32.256%	6.666%	9.161%	0.058%	5.160%	60.229%	53.096%	27.448%	37.161%
	$se\left(\widehat{R^2}\right)$	0.1843	0.3325	0.2388	0.4030	0.0478	0.1374	0.3117	0.2369	0.4129	0.7350
GLS	CSR R^2	8.124%	13.553%	8.692%	0.098%	0.004%	2.325%	29.792%	11.466%	3.605%	22.506%
	$se\left(\widehat{R^2}\right)$	0.0755	0.1156	0.1556	0.0120	0.0034	0.0487	0.1724	0.0348	0.0719	0.1974
GLS	H_0 : CSR $R^2=1$	0.0166	0.0597	0.0068	0.0068	0.0085	0.0094	0.1555	0.0482	0.0102	0.1975
	H_0 : CSR $R^2=0$	0.0603	0.0312	0.2769	0.8659	0.9815	0.3525	0.0007	0.0029	0.3278	0.0172
		Kroencke - unfiltered NIPA nondurables and services					**Kroencke - unfiltered NIPA nondurables**				
OLS	$\gamma_{0,j}$	2.0933	1.6815	1.4909	1.2234	1.7708	2.1902	1.5257	1.1248	0.6991	0.1902
	MR t-statistic	4.3007	3.0010	1.7897	0.7672	0.5881	4.3626	3.4259	1.1744	0.5362	0.9565
	boot p-val	0.0380	0.0805	0.1505	0.4800	0.5920	0.0160	0.0360	0.3890	0.5850	0.5780
OLS	$\gamma_{1,j}$	0.1131	0.2964	0.2744	0.1650	0.0707	0.1028	0.4455	0.5614	0.2560	0.2526
	MR t-statistic	0.4140	1.7745	0.8083	0.6025	0.1941	0.2873	2.2353	1.2904	1.0444	13.5544
	boot p-val	0.6055	0.2140	0.4530	0.5265	0.8320	0.7575	0.1390	0.2995	0.3190	0.0015
GLS	$\gamma_{1,j}$	-0.3500	0.3032	0.4035	-0.0237	0.0576	-0.9027	0.3164	0.2554	-0.0139	0.0822
	MR t-statistic	-1.4650	2.4084	1.2866	-0.2553	0.6109	-3.0093	1.9503	0.6055	-0.1500	0.8575
	boot p-val	0.3040	0.0795	0.5015	0.7765	0.6465	0.3850	0.1290	0.6360	0.8765	0.5745
OLS	CSR R^2	3.073%	39.912%	11.227%	11.320%	4.184%	0.888%	51.611%	20.690%	26.240%	65.114%
	$se\left(\widehat{R^2}\right)$	0.1336	0.3398	0.2642	0.3237	0.4187	0.0768	0.3320	0.3004	0.4040	0.2450
GLS	CSR R^2	6.797%	16.073%	9.499%	0.199%	4.307%	23.019%	10.366%	1.845%	0.067%	7.089%
	$se\left(\widehat{R^2}\right)$	0.0510	0.1226	0.1504	0.0150	0.1063	0.1526	0.0955	0.0731	0.0090	0.1372
GLS	H_0 : CSR $R^2=1$	0.0043	0.0470	0.0081	0.0052	0.0005	0.0760	0.0531	0.0004	0.0100	0.0164
	H_0 : CSR $R^2=0$	0.0709	0.0151	0.2177	0.7961	0.5071	0.0009	0.0347	0.5457	0.8800	0.3510

Notes: This table reports estimates for the zero-beta excess return (OLS) and the price of frequency-specific beta risk (OLS and GLS) for consumption growth (innovations: first-difference, the factor is priced at $j=4$ in Bandi and Tamoni, 2013 and at $j>4$ in the IA of Kang et al., 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) t-statistics using a HAC variance-covariance matrix. The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing H_0 : $R^2=1$) and H_0 : CSR $R^2=0$ (imposing H_0 : $\gamma_{1,j}=0$) for the GLS case. The beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and the filter is LA(8). The test assets are the 25 FF size and investment value-weighted portfolios and the data are quarterly from 1963:Q3 to 2018:Q4. We use four different measures of consumption growth from Kroencke (2017), namely: NIPA nondurables and services, three-year NIPA consumption (Parker and Julliard, 2005), unfiltered NIPA nondurables and services and unfiltered NIPA nondurables.

Table IA.22: Spectral Zoo: Consumption Growth from Bandi and Tamoni (2013) and Kang et al. (2017)

size and operating profitability portfolios: 1963:Q3 - 2018:Q4

	Frequency in cycles per period	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$
		NIPA nondurables and services					**Parker and Julliard - consumption growth**				
OLS	$\gamma_{0,j}$	1.5028	1.3274	1.5167	2.8741	3.4322	2.2204	2.3159	1.5315	1.5266	2.6027
	MR t-statistic	2.4597	1.9280	0.8522	1.7496	3.3756	4.2616	5.2894	2.1725	1.6747	1.5429
	boot p-val	0.1985	0.2110	0.4860	0.2615	0.0550	0.0060	0.0020	0.1290	0.2050	0.1825
OLS	$\gamma_{1,j}$	0.1730	0.1204	0.1043	-0.0690	-0.1107	0.4026	0.1256	0.2330	0.3596	-0.1505
	MR t-statistic	1.5714	1.5807	0.3909	-0.4029	-1.7883	1.5973	0.9674	1.4986	1.5548	-0.2608
	boot p-val	0.2185	0.1415	0.7310	0.7400	0.2995	0.3910	0.5025	0.1950	0.1810	0.8140
GLS	$\gamma_{1,j}$	0.0119	0.0826	-0.0392	-0.0030	0.0260	-0.0612	0.0514	0.1617	0.0485	-0.0209
	MR t-statistic	0.1809	1.6382	-0.3618	-0.0491	0.7689	-0.5439	1.1005	2.0139	0.4747	-0.1871
	boot p-val	0.8650	0.1885	0.7135	0.9510	0.5710	0.6165	0.1925	0.3620	0.6560	0.8330
OLS	CSR R^2	38.993%	45.036%	4.220%	6.429%	39.437%	23.535%	11.648%	45.176%	24.981%	4.226%
	$se\left(\widehat{R^2}\right)$	0.2806	0.3379	0.2189	0.3446	0.4416	0.2373	0.2713	0.3461	0.4838	0.2893
GLS	CSR R^2	0.099%	10.157%	0.741%	0.014%	4.158%	1.079%	4.405%	27.474%	1.259%	0.274%
	$se\left(\widehat{R^2}\right)$	0.0113	0.1265	0.0385	0.0041	0.1016	0.0257	0.0799	0.2023	0.0552	0.0277
GLS	H_0 : CSR $R^2=1$	0.0685	0.0894	0.0620	0.0747	0.0927	0.0791	0.0822	0.4784	0.0802	0.0530
	H_0 : CSR $R^2=0$	0.8568	0.1594	0.7057	0.9609	0.4469	0.5800	0.2962	0.0188	0.6246	0.8472
		Kroencke - unfiltered NIPA nondurables and services					**Kroencke - unfiltered NIPA nondurables**				
OLS	$\gamma_{0,j}$	1.5421	1.6333	1.0329	1.9286	3.2414	1.6697	1.4164	1.5122	1.4322	1.8961
	MR t-statistic	2.6772	2.7199	0.8325	0.9237	2.9066	2.4378	2.8835	1.2391	0.7885	1.0068
	boot p-val	0.1830	0.1030	0.4935	0.5745	0.0840	0.1765	0.0470	0.3275	0.5745	0.3720
OLS	$\gamma_{1,j}$	0.3603	0.2845	0.4059	0.0414	-0.1513	0.5185	0.4595	0.3314	0.1242	0.0376
	MR t-statistic	1.5475	1.6013	0.9627	0.1400	-1.6531	1.2358	2.3219	0.5922	0.5031	0.1782
	boot p-val	0.2340	0.2295	0.4005	0.9035	0.3810	0.3875	0.0810	0.5710	0.6845	0.8500
GLS	$\gamma_{1,j}$	0.0194	0.1319	0.0630	0.0275	0.0221	0.0396	0.2164	0.0402	0.0414	0.0318
	MR t-statistic	0.1411	1.0558	0.2415	0.2709	0.4784	0.1500	1.7016	0.1608	0.4672	0.6435
	boot p-val	0.9060	0.3385	0.8055	0.7630	0.7220	0.8495	0.1255	0.8580	0.6080	0.6020
OLS	CSR R^2	32.755%	35.350%	19.732%	0.921%	39.697%	24.204%	58.487%	8.028%	7.268%	1.115%
	$se\left(\widehat{R^2}\right)$	0.2863	0.3461	0.3651	0.1292	0.4554	0.2942	0.3307	0.2066	0.2956	0.1395
GLS	CSR R^2	0.060%	5.284%	0.418%	0.458%	1.772%	0.098%	10.109%	0.161%	0.999%	2.280%
	$se\left(\widehat{R^2}\right)$	0.0086	0.0986	0.0308	0.0231	0.0718	0.0096	0.0936	0.0185	0.0306	0.0682
GLS	H_0 : CSR $R^2=1$	0.0651	0.0673	0.0667	0.0866	0.0733	0.0639	0.1074	0.0724	0.0867	0.0809
	H_0 : CSR $R^2=0$	0.8879	0.3315	0.8067	0.7854	0.6262	0.8699	0.1149	0.8733	0.6486	0.5209

Notes: This table reports estimates for the zero-beta excess return (OLS) and the price of frequency-specific beta risk (OLS and GLS) for consumption growth (innovations: first-difference, the factor is priced at $j=4$ in Bandi and Tamoni, 2013 and at $j>4$ in the IA of Kang et al., 2017). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) t-statistics using a HAC variance-covariance matrix. The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0: R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0: \gamma_{1,j}=0$) for the GLS case. The beta is defined based on the MODWT MRA indirect method (ignoring boundary effects) and the filter is LA(8). The test assets are the 25 FF size and op. profitability value-weighted portfolios and the data are quarterly from 1963:Q3 to 2018:Q4. We use four different measures of consumption growth from Kroencke (2017), namely: NIPA nondurables and services, three-year NIPA consumption (Parker and Julliard, 2005), unfiltered NIPA nondurables and services and unfiltered NIPA nondurables.

Table IA.23a: MODWT MRA with Boundary Independent Coefficients Only

	Frequency in cycles per period	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$
		Panel A: Macro Uncertainty with ME and BE-ME					Panel B: Macro Uncertainty with ME and INV				
OLS	$\gamma_{0,j}$	0.3693	0.7820	0.7954	0.6680	0.0786	0.5285	0.8679	0.9421	0.9541	0.0594
	MR t-statistic	1.4561	2.0868	2.7312	0.6844	0.2221	1.8947	3.4546	4.7965	1.2459	0.1529
	boot p-val	0.3200	0.0295	0.0055	0.4685	0.8770	0.2160	0.0005	0.0005	0.2770	0.9005
OLS	$\gamma_{1,j}$	-0.3084	0.0643	0.2205	-0.0003	-0.5560	-0.1792	0.1385	0.4820	0.2041	-0.5803
	MR t-statistic	-1.1338	0.1762	0.4539	-0.0004	-1.4659	-0.6481	0.5179	1.5450	0.3336	-1.3846
	boot p-val	0.2860	0.8550	0.6015	0.9995	0.2165	0.6660	0.5240	0.2205	0.7530	0.2105
GLS	$\gamma_{1,j}$	-0.0276	0.1784	0.0795	0.2137	-0.1446	-0.0177	0.0637	0.2233	0.1399	-0.1926
	MR t-statistic	-0.2570	0.9624	0.4984	1.5618	-0.7132	-0.1293	0.4543	1.0562	0.5307	-0.7549
	boot p-val	0.7795	0.3465	0.5450	0.2945	0.5305	0.9480	0.6570	0.3045	0.6820	0.4640
OLS	CSR R^2	15.957%	0.421%	2.489%	0.000%	32.279%	5.635%	2.735%	16.905%	3.693%	38.129%
	$se\left(\widehat{R^2}\right)$	0.1901	0.0449	0.1260	0.0004	0.4573	0.1779	0.0942	0.1637	0.2166	0.4270
GLS	CSR R^2	0.179%	4.301%	0.686%	12.580%	3.312%	0.056%	0.653%	3.925%	2.073%	2.798%
	$se\left(\widehat{R^2}\right)$	0.0111	0.0815	0.0274	0.1618	0.0791	0.0085	0.0282	0.0564	0.0765	0.0732
GLS	H_0 : CSR $R^2 = 1$	0.0035	0.0034	0.0021	0.0257	0.0262	0.0019	0.0008	0.0006	0.0045	0.0071
	H_0 : CSR $R^2 = 0$	0.7920	0.2954	0.6362	0.1551	0.4214	0.8970	0.6459	0.2487	0.5933	0.4501
		Panel C: Macro Uncertainty with ME and OP					Panel D: Financial Uncertainty with ME and BE-ME				
OLS	$\gamma_{0,j}$	0.2338	0.3372	0.4696	0.1900	0.2651	0.5377	0.6446	0.6379	0.9624	0.3615
	MR t-statistic	0.7444	0.9218	1.4109	0.2128	0.7236	1.6302	1.7003	1.7191	1.2473	1.0698
	boot p-val	0.5985	0.3220	0.1265	0.8325	0.5455	0.1565	0.1410	0.1270	0.1975	0.2950
OLS	$\gamma_{1,j}$	-0.4142	-0.3200	-0.3341	-0.3232	-0.3556	-0.3186	-0.0796	-0.0490	0.3419	-0.4805
	MR t-statistic	-1.0438	-0.9136	-0.5707	-0.4485	-0.8713	-0.5140	-0.1598	-0.0993	0.3735	-0.8043
	boot p-val	0.3195	0.3020	0.5775	0.6530	0.4085	0.6705	0.8990	0.9245	0.6830	0.4165
GLS	$\gamma_{1,j}$	0.0242	0.2048	0.0395	0.0742	-0.0115	0.3408	0.2753	0.0540	0.6558	-0.0632
	MR t-statistic	0.1844	1.3461	0.2644	0.4996	-0.0601	0.8771	0.7539	0.1778	2.8507	-0.1804
	boot p-val	0.8520	0.3630	0.7600	0.6950	0.9440	0.3785	0.4610	0.8200	0.0630	0.8640
OLS	CSR R^2	38.753%	11.638%	9.927%	14.835%	15.830%	4.038%	0.336%	0.119%	3.195%	11.128%
	$se\left(\widehat{R^2}\right)$	0.2452	0.2289	0.3135	0.6505	0.3551	0.1629	0.0420	0.0244	0.1909	0.2348
GLS	CSR R^2	0.173%	13.874%	0.352%	2.410%	0.030%	3.184%	1.995%	0.096%	23.538%	0.182%
	$se\left(\widehat{R^2}\right)$	0.0176	0.2139	0.0265	0.1011	0.0103	0.0744	0.0527	0.0092	0.1576	0.0184
GLS	H_0 : CSR $R^2 = 1$	0.0733	0.2424	0.0751	0.2139	0.1835	0.0002	0.0005	0.0021	0.0455	0.0130
	H_0 : CSR $R^2 = 0$	0.8501	0.1700	0.7920	0.6457	0.9519	0.3979	0.4762	0.8555	0.0084	0.8527

Notes: This table reports additional results for a beta decomposition based on the MODWT MRA **indirect** method using boundary independent coefficients only - the filter is LA(8). In Panel A the priced factor is macro uncertainty (innovations: first-difference, $h = 1$) with size and book-to-market portfolios, in Panel B macro uncertainty with size and investment portfolios, in Panel C macro uncertainty with size and operating profitability portfolios and in Panel D the priced factor is financial uncertainty (innovations: first-difference, $h = 1$) with size and book-to-market portfolios. Inference is based on Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size).

Table IA.23b: MODWT MRA with Boundary Independent Coefficients Only

	Frequency in cycles per period	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	>4 $\left[\frac{1}{32},0\right]$	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	>4 $\left[\frac{1}{32},0\right]$
		Panel E: Financial Uncertainty with ME and INV					Panel F: Financial Uncertainty with ME and OP				
OLS	$\gamma_{0,j}$	0.4018	0.6309	0.6802	0.9328	0.3927	0.2855	-0.0018	0.3605	0.3669	0.3269
	MR t-statistic	1.4392	1.8995	2.3273	2.7167	1.1380	0.9227	-0.0046	1.0574	0.4773	0.9348
	boot p-val	0.2420	0.1395	0.0445	0.0190	0.2755	0.5295	0.9965	0.3875	0.5735	0.3600
OLS	$\gamma_{1,j}$	-0.6064	-0.1047	-0.0038	0.3098	-0.4623	-0.7421	-0.8024	-0.3457	-0.3184	-0.5009
	MR t-statistic	-0.8454	-0.2236	-0.0088	0.6862	-0.8372	-1.0298	-1.3489	-0.8048	-0.3188	-0.9607
	boot p-val	0.4570	0.8640	0.9935	0.4310	0.3945	0.4555	0.2300	0.4810	0.7420	0.3005
GLS	$\gamma_{1,j}$	-0.3368	0.3464	0.1324	0.6981	-0.1446	0.2624	0.1217	0.1238	0.1318	-0.0202
	MR t-statistic	-0.6030	0.7518	0.3849	2.0112	-0.3621	0.6645	0.4201	0.4133	0.4978	-0.0705
	boot p-val	0.6285	0.4010	0.6110	0.2520	0.7380	0.5615	0.6780	0.6280	0.7190	0.9420
OLS	CSR R^2	16.009%	0.645%	0.001%	3.373%	13.492%	25.706%	34.395%	9.633%	4.219%	19.132%
	$se\left(\widehat{R^2}\right)$	0.2925	0.0592	0.0021	0.1263	0.2676	0.3143	0.3584	0.2280	0.2758	0.2811
GLS	CSR R^2	1.952%	2.707%	0.410%	14.849%	0.554%	2.879%	0.918%	0.879%	1.849%	0.037%
	$se\left(\widehat{R^2}\right)$	0.0692	0.0750	0.0212	0.1234	0.0299	0.0864	0.0415	0.0369	0.0794	0.0106
GLS	H_0 : CSR $R^2=1$	0.0027	0.0007	0.0004	0.0183	0.0019	0.0831	0.0667	0.0836	0.2254	0.1740
	H_0 : CSR $R^2=0$	0.5698	0.4740	0.7038	0.0131	0.7171	0.5078	0.6804	0.6842	0.6386	0.9435
		Panel G: IPG with ME and BE-ME					Panel H: IPG with ME and INV				
OLS	$\gamma_{0,j}$	0.3753	1.0221	0.6569	1.0994	0.7780	0.4165	1.1818	0.6244	1.0341	0.6209
	MR t-statistic	1.4016	3.0131	3.0487	3.5924	1.4914	1.3097	2.6949	3.3083	3.6372	1.3661
	boot p-val	0.2810	0.0055	0.0170	0.0015	0.1350	0.3680	0.0345	0.0115	0.0000	0.2025
OLS	$\gamma_{1,j}$	-0.7140	0.1740	0.0052	-0.0285	-0.0046	-0.6763	0.2508	0.0141	-0.0229	0.0023
	MR t-statistic	-1.4440	0.9332	0.1122	-1.1876	-0.1871	-1.2424	0.8854	0.3584	-0.9261	0.1076
	boot p-val	0.2470	0.3105	0.8965	0.3850	0.8325	0.5505	0.3855	0.7190	0.4870	0.9205
GLS	$\gamma_{1,j}$	-0.1134	0.2028	-0.0164	-0.0391	-0.0021	0.0593	0.2038	-0.0380	-0.0480	-0.0105
	MR t-statistic	-0.4421	2.1035	-0.3918	-2.5928	-0.1871	0.2096	1.5536	-1.1778	-2.1347	-0.6870
	boot p-val	0.6690	0.2405	0.6425	0.0975	0.7995	0.8765	0.3410	0.2110	0.2465	0.5200
OLS	CSR R^2	26.956%	23.025%	0.167%	24.282%	0.702%	15.186%	24.924%	1.493%	14.411%	0.198%
	$se\left(\widehat{R^2}\right)$	0.1817	0.3824	0.0294	0.3319	0.0780	0.2145	0.4814	0.0808	0.3001	0.0385
GLS	CSR R^2	0.809%	23.929%	0.592%	28.627%	0.164%	0.184%	12.789%	3.301%	25.076%	2.467%
	$se\left(\widehat{R^2}\right)$	0.0344	0.1499	0.0298	0.1958	0.0175	0.0174	0.1587	0.0485	0.1637	0.0725
GLS	H_0 : CSR $R^2=1$	0.0043	0.0472	0.0017	0.1239	0.0097	0.0018	0.0296	0.0004	0.0468	0.0025
	H_0 : CSR $R^2=0$	0.6625	0.0018	0.6876	0.0102	0.8521	0.8324	0.1107	0.2137	0.0046	0.4849

Notes: This table reports additional results for a beta decomposition based on the MODWT MRA **indirect** method using boundary independent coefficients only - the filter is LA(8). In Panel E the priced factor is financial uncertainty (innovations: first-difference, $h=1$) with size and investment portfolios, in Panel F financial uncertainty with size and operating profitability portfolios, in Panel G the priced factor is industrial production growth (IPG) (innovations: first-difference) with size and book-to-market portfolios and in Panel H IPG with size and investment portfolios. Inference is based on Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size).

Table IA.23c: MODWT MRA with Boundary Independent Coefficients Only

	Frequency in cycles per period	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$	$j=1$ $[\frac{1}{2},\frac{1}{4}]$	2 $[\frac{1}{4},\frac{1}{8}]$	3 $[\frac{1}{8},\frac{1}{16}]$	4 $[\frac{1}{16},\frac{1}{32}]$	>4 $[\frac{1}{32},0]$
		Panel I: IPG with ME and OP					Panel J: Volatility of IPG with ME and BE-ME				
OLS	$\gamma_{0,j}$	0.4374	1.0727	0.4183	0.4966	0.2853	0.7095	0.8760	0.8774	1.4107	0.1483
	MR t-statistic	1.3991	3.3786	1.7444	0.9808	0.6799	4.9241	3.6133	3.1349	4.7216	0.2571
	boot p-val	0.2450	0.0125	0.1070	0.5045	0.6385	0.0000	0.0000	0.0020	0.0145	0.8460
OLS	$\gamma_{1,j}$	-0.5489	0.2076	0.0559	0.0088	0.0145	-0.0084	0.0260	0.0420	0.0313	-0.0080
	MR t-statistic	-0.8697	1.0748	1.4697	0.2355	0.8379	-0.0957	0.7051	1.1537	2.3711	-0.7459
	boot p-val	0.6490	0.2625	0.2670	0.8645	0.4685	0.9360	0.4545	0.3275	0.1335	0.4475
GLS	$\gamma_{1,j}$	0.3795	0.0955	0.0036	-0.0162	-0.0030	-0.0361	0.0104	0.0308	0.0163	-0.0013
	MR t-statistic	1.7754	1.6371	0.0921	-1.3029	-0.3980	-0.7693	0.5209	2.1583	2.4661	-0.2821
	boot p-val	0.2120	0.2995	0.9380	0.2420	0.5765	0.6765	0.5500	0.3515	0.1705	0.7230
OLS	CSR R^2	12.801%	26.414%	34.535%	1.658%	11.926%	0.076%	9.194%	36.226%	36.958%	12.532%
	$se\left(\widehat{R^2}\right)$	0.2382	0.4833	0.3284	0.1226	0.2695	0.0144	0.2355	0.2411	0.3525	0.2562
GLS	CSR R^2	16.621%	9.839%	0.068%	8.517%	0.755%	2.323%	0.929%	14.156%	23.174%	0.757%
	$se\left(\widehat{R^2}\right)$	0.1523	0.1124	0.0145	0.1305	0.0343	0.0584	0.0315	0.1554	0.1447	0.0534
GLS	H_0 : CSR $R^2 = 1$	0.2489	0.1770	0.0721	0.2453	0.1352	0.0028	0.0032	0.0528	0.1059	0.0119
	H_0 : CSR $R^2 = 0$	0.0381	0.1353	0.9266	0.2317	0.6900	0.4589	0.6190	0.0171	0.0142	0.7509
		Panel K: Volatility of IPG with ME and INV					Panel L: Volatility of IPG with ME and OP				
OLS	$\gamma_{0,j}$	0.7031	1.1181	0.8633	1.2246	0.4842	0.6518	0.5187	0.6463	0.9890	1.0532
	MR t-statistic	4.1137	2.4363	3.7726	5.3215	1.0313	3.1235	1.0957	2.6326	1.7197	2.8849
	boot p-val	0.0005	0.0330	0.0020	0.0015	0.3315	0.0075	0.3020	0.0090	0.2055	0.0130
OLS	$\gamma_{1,j}$	0.0461	0.0597	0.0357	0.0234	-0.0030	0.0815	-0.0250	-0.0005	0.0148	0.0065
	MR t-statistic	0.4161	0.7496	1.2768	1.7334	-0.3318	0.9306	-0.3348	-0.0111	0.5512	0.9075
	boot p-val	0.6845	0.4875	0.3475	0.1880	0.6745	0.4515	0.7590	0.9910	0.5905	0.3105
GLS	$\gamma_{1,j}$	-0.0102	0.0310	0.0168	0.0096	-0.0040	0.0136	0.0000	0.0142	0.0065	0.0021
	MR t-statistic	-0.2920	1.2770	1.3977	1.0108	-0.7071	0.4279	0.0020	1.1432	1.2770	0.7531
	boot p-val	0.7755	0.1930	0.3075	0.5530	0.5240	0.6675	0.9990	0.3690	0.3515	0.4430
OLS	CSR R^2	2.886%	16.994%	26.285%	34.136%	1.504%	8.855%	2.307%	0.003%	9.801%	10.290%
	$se\left(\widehat{R^2}\right)$	0.1250	0.4051	0.2255	0.2907	0.0970	0.1317	0.1895	0.0057	0.3818	0.1973
GLS	CSR R^2	0.196%	5.438%	5.524%	5.161%	3.197%	0.688%	0.000%	7.015%	7.502%	3.605%
	$se\left(\widehat{R^2}\right)$	0.0133	0.0717	0.0749	0.0766	0.0868	0.0320	0.0003	0.1059	0.1242	0.0923
GLS	H_0 : CSR $R^2 = 1$	0.0020	0.0032	0.0013	0.0041	0.0022	0.0818	0.0867	0.1273	0.2968	0.1899
	H_0 : CSR $R^2 = 0$	0.7691	0.1885	0.1049	0.2115	0.4690	0.6757	0.9984	0.1652	0.2867	0.4785

Notes: This table reports additional results for a beta decomposition based on the MODWT MRA **indirect** method using boundary independent coefficients only - the filter is LA(8). In Panel I the priced factor is industrial production growth (IPG) (innovations: first-difference) with size and operating profitability portfolios, in Panel J the priced factor is the volatility of IPG (innovations: first-difference) with size and book-to-market portfolios, in Panel K volatility of IPG with size and investment portfolios and in Panel L volatility of IPG with size and operating profitability portfolios. Inference is based on Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size).

Table IA.24: Spectral Zoo: FF3 from Kang et al. (2017)

	Frequency in cycles	$j=1$ $\left[\frac{1}{2},\frac{1}{4}\right]$	2 $\left[\frac{1}{4},\frac{1}{8}\right]$	3 $\left[\frac{1}{8},\frac{1}{16}\right]$	4 $\left[\frac{1}{16},\frac{1}{32}\right]$	5 $\left[\frac{1}{32},\frac{1}{64}\right]$	6 $\left[\frac{1}{64},\frac{1}{128}\right]$	>6 $\left[\frac{1}{128},0\right]$
		size and book-to-market portfolios						
OLS	$\lambda_{0,j}$	1.121	0.865	0.945	1.235	1.205	0.958	1.016
	MR t-stat	3.658	2.909	2.917	4.426	4.484	4.274	2.234
	boot p-val	0.001	0.010	0.021	0.002	0.000	0.002	0.056
	$\lambda_{1,j}^{MKT}$	-0.054	-0.062	-0.276	-0.732	-1.097	-1.040	-1.649
	MR t-stat	-1.471	-0.976	-1.415	-2.141	-2.151	-1.372	-0.556
	boot p-val	0.089	0.308	0.197	0.033	0.050	0.139	0.600
	$\lambda_{2,j}^{SMB}$	0.061	0.122	0.405	1.043	1.573	0.375	0.133
	MR t-stat	1.953	1.698	1.971	2.360	2.019	0.298	0.195
	boot p-val	0.092	0.122	0.062	0.020	0.076	0.768	0.842
	$\lambda_{3,j}^{HML}$	0.072	0.153	0.221	0.481	0.273	0.785	0.432
	MR t-stat	1.592	1.953	1.834	1.692	0.857	0.801	0.224
	boot p-val	0.064	0.042	0.066	0.033	0.274	0.434	0.873
GLS	$\lambda_{0,j}$	1.255	1.113	1.015	1.188	1.112	0.898	0.957
	MR t-stat	4.447	4.418	3.716	5.109	5.123	4.764	3.052
	boot p-val	0.005	0.005	0.013	0.001	0.000	0.003	0.020
	$\lambda_{1,j}^{MKT}$	-0.066	-0.096	-0.234	-0.573	-0.673	-0.733	-1.309
	MR t-stat	-1.966	-1.695	-1.296	-1.913	-1.685	-1.094	-0.726
	boot p-val	0.098	0.068	0.241	0.054	0.096	0.252	0.591
	$\lambda_{2,j}^{SMB}$	0.082	0.130	0.355	0.894	1.042	-0.498	-0.860
	MR t-stat	2.686	1.856	1.771	2.085	1.663	-0.357	-1.223
	boot p-val	0.020	0.062	0.076	0.055	0.120	0.720	0.263
	$\lambda_{3,j}^{HML}$	0.038	0.130	0.240	0.411	0.272	1.249	0.063
	MR t-stat	0.861	1.633	1.943	1.631	0.895	1.232	0.051
	boot p-val	0.361	0.120	0.061	0.064	0.228	0.220	0.973
OLS	CSR R^2	59.61%	53.27%	57.09%	66.70%	70.98%	67.73%	54.48%
	$se\left(\widehat{R^2}\right)$	0.237	0.240	0.252	0.194	0.172	0.208	0.231
GLS	CSR R^2	19.71%	13.22%	16.91%	26.15%	24.58%	20.31%	14.12%
	$se\left(\widehat{R^2}\right)$	0.124	0.118	0.157	0.166	0.153	0.197	0.148
	$H_0 : R^2 = 1$	0.008	0.000	0.004	0.020	0.013	0.005	0.001
	$H_0 : R^2 = 0$	0.009	0.046	0.071	0.019	0.041	0.280	0.399

Notes: This table reports estimates for the zero-beta excess return and the price of frequency-specific covariance risk (OLS and GLS case) for the Fama-French 3 factor model (the model performs better at $j = 3$ in Kang et al., 2017). Inference is based on Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2 = 1$ (imposing $H_0 : R^2 = 1$) and H_0 : CSR $R^2 = 0$ (imposing $H_0 : \lambda_{1,j} = 0$) for the GLS case. The data are monthly from July 1963 through December 2020.

Table IA.25: Spectral Zoo: MKT from Bandi et al. (2021) - Wavelet Approach

	Frequency in cycles per period	$j=4$ $\frac{1}{16},\frac{1}{32}$	5 $\frac{1}{32},\frac{1}{64}$	6 $\frac{1}{64}\cdot\frac{1}{128}$	>6 $\frac{1}{128},0$	4 $\frac{1}{16},\frac{1}{32}$	5 $\frac{1}{32},\frac{1}{64}$	6 $\frac{1}{64}\cdot\frac{1}{128}$	>6 $\frac{1}{128},0$	4 $\frac{1}{16},\frac{1}{32}$	5 $\frac{1}{32},\frac{1}{64}$	6 $\frac{1}{64}\cdot\frac{1}{128}$	>6 $\frac{1}{128},0$
		Beta decomposition based on **MODWT MRA**, **Indirect**											
		ME and BE/ME portfolios				**ME and INV portfolios**				**ME and OP portfolios**			
OLS	$\gamma_{0,j}$	0.7986	0.9120	1.3311	1.1235	0.7396	0.7965	1.2455	1.1100	0.4115	0.6966	1.1642	1.0988
	FM t-statistic	2.6693	3.3902	5.4997	4.7728	3.3143	3.6138	5.5199	4.5728	1.5804	3.1773	4.3939	3.6852
	MR t-statistic	2.2168	2.6800	4.0863	4.1027	2.7420	2.7359	4.1682	5.2797	1.0519	1.6967	3.3705	3.8250
	boot p-val	0.0740	0.0155	0.0070	0.0005	0.0305	0.0210	0.0040	0.0005	0.4765	0.2860	0.0295	0.0115
OLS	$\gamma_{1,j}$	-0.0254	-0.1037	-0.5277	-0.4837	0.0102	-0.0235	-0.4409	-0.4363	0.1592	0.0188	-0.3883	-0.4633
	FM t-statistic	-0.1410	-0.5328	-4.3488	-3.3440	0.0679	-0.1321	-3.6243	-3.2733	0.9504	0.1115	-3.1488	-2.4510
	MR t-statistic	-0.1137	-0.3951	-2.5477	-1.6061	0.0551	-0.0972	-2.1238	-1.9369	0.7217	0.0688	-1.6317	-1.5470
	boot p-val	0.9240	0.7210	0.1175	0.1660	0.9595	0.9275	0.0940	0.0975	0.5935	0.9540	0.2425	0.2875
GLS	$\gamma_{1,j}$	-0.2364	-0.2711	-0.1945	-0.1805	-0.1463	-0.0943	-0.3481	-0.2686	-0.0652	-0.1013	-0.1462	-0.0337
	FM t-statistic	-2.3098	-2.6373	-3.0716	-2.0311	-1.4261	-0.8729	-3.8348	-3.0230	-0.6459	-1.0048	-2.0219	-0.3684
	MR t-statistic	-1.3361	-1.4968	-1.6812	-0.8875	-0.8911	-0.4443	-2.0480	-1.3360	-0.4415	-0.6280	-1.0933	-0.1897
	boot p-val	0.2035	0.0320	0.1190	0.4335	0.4075	0.6700	0.1455	0.3055	0.6435	0.3695	0.3035	0.8400
OLS	CSR R^2	0.233%	3.159%	55.587%	53.870%	0.048%	0.164%	56.100%	58.648%	9.271%	0.103%	64.550%	55.679%
	$se\left(\widehat{R^2}\right)$	0.0415	0.1477	0.2961	0.2335	0.0171	0.0330	0.2347	0.2221	0.2281	0.0283	0.2958	0.4132
GLS	CSR R^2	6.880%	8.969%	12.166%	5.320%	2.225%	0.834%	16.086%	9.996%	0.989%	2.394%	9.694%	0.322%
	$se\left(\widehat{R^2}\right)$	0.1006	0.1119	0.1649	0.1048	0.0425	0.0367	0.1641	0.1372	0.0454	0.0546	0.1882	0.0341
GLS	H_0 : CSR $R^2=1$	0.0000	0.0030	0.0045	0.0030	0.0004	0.0012	0.0052	0.0027	0.0821	0.0740	0.0900	0.0585
	H_0 : CSR $R^2=0$	0.2019	0.1811	0.1592	0.3069	0.3579	0.6517	0.0545	0.1726	0.6625	0.5230	0.3434	0.8528

Notes: This table reports estimates for the zero-beta excess return (OLS case) and the price of frequency-specific beta risk (OLS and GLS case) for the MKT factor (priced at $j=6$ in Bandi et al., 2021). Inference is based on Fama-MacBeth (1973) and Kan et al. (2013) misspecification robust t-statistics using a HAC variance-covariance matrix (see Newey and West, 1994). boot p-val (bootstrap p-value) is constructed by bootstrapping the MR covariance matrix in Kan et al. (2013) based on 2,000 replications (using the circular block bootstrap and the Politis and White, 2004 estimator for the block size). The table also presents the cross-sectional R^2 and its standard error (OLS and GLS case) along with the specification tests of H_0 : CSR $R^2=1$ (imposing $H_0: R^2=1$) and H_0 : CSR $R^2=0$ (imposing $H_0:\gamma_1=0$) for the GLS case. The test assets are the 25 FF size and book-to-market, size and investment & size and operating profitability value-weighted portfolios. The beta decomposition is based on the MODWT MRA indirect method (ignoring boundary effects). The filter used is LA(8). To preserve space we only report results for $j=4,5,6>6$.

Table IA.26: Identification Failure: Rank Tests for Useless and Useful Factors

		N = 1024								N = 256							
Panel A		Beta decomposition based on **MODWT MRA**, **Indirect**															
		$j=1$	2	3	4	5	>5	6	>6	$j=1$	2	3	4	5	>5	6	>6
		Useless Factor and Actual Returns - Table 4															
rank test	$\mathcal{J}_1$	0.105	0.068	0.077	0.089	0.179	0.244	0.180	0.156	0.287	0.227	0.267	0.348	0.329	0.385	0.375	0.219
	$\mathcal{J}_2$	0.064	0.035	0.037	0.050	0.109	0.136	0.134	0.131	0.053	0.036	0.064	0.129	0.107	0.160	0.205	0.072
		Useless Factor and Simulated Returns - Table IA.4															
rank test	$\mathcal{J}_1$	0.095	0.102	0.101	0.119	0.134	0.130	0.094	0.090	0.317	0.305	0.297	0.296	0.272	0.263	0.241	0.208
	$\mathcal{J}_2$	0.046	0.050	0.050	0.055	0.045	0.045	0.055	0.048	0.051	0.049	0.052	0.051	0.049	0.052	0.048	0.053
Panel B		Beta decomposition based on **MODWPT MRA**, **Indirect**															
		$n=0$	1	2	3	4	5	6	7	$n=0$	1	2	3	4	5	6	7
		Useless Factor and Actual Returns - Table 5															
rank test	$\mathcal{J}_1$	0.119	0.077	0.081	0.061	0.061	0.104	0.125	0.130	0.460	0.517	0.476	0.413	0.409	0.455	0.594	0.488
	$\mathcal{J}_2$	0.080	0.037	0.043	0.032	0.032	0.059	0.072	0.083	0.079	0.077	0.055	0.048	0.035	0.034	0.114	0.077
		Useless Factor and Simulated Returns - Table IA.5															
rank test	$\mathcal{J}_1$	0.095	0.101	0.105	0.100	0.098	0.097	0.103	0.098	0.503	0.496	0.506	0.497	0.502	0.505	0.503	0.496
	$\mathcal{J}_2$	0.042	0.050	0.053	0.049	0.052	0.049	0.050	0.047	0.049	0.048	0.047	0.049	0.050	0.054	0.052	0.053

Beta decomposition based on			**MODWT MRA**, **Indirect**					**MODWPT MRA**, **Indirect**							
			$j=1$	2	3	4	>4	$n=0$	1	2	3	4	5	6	7
Panel C			Useful Factor Priced at High Frequencies - Table 6a, Panel A and Table IA.6a, Panel A												
rank test	$\mathcal{J}_1$	**N = 1024**	1.000	0.107	0.151	0.321	0.519	0.443	0.151	0.117	0.094	0.075	0.059	0.115	1.000
	$\mathcal{J}_2$		1.000	0.058	0.073	0.208	0.420	0.337	0.073	0.057	0.047	0.031	0.024	0.050	1.000
	$\mathcal{J}_1$	**N = 256**	1.000	0.519	0.578	0.984	0.995	0.785	0.578	0.563	0.605	0.658	0.284	0.392	1.000
	$\mathcal{J}_2$		1.000	0.106	0.071	0.112	0.193	0.207	0.071	0.076	0.159	0.163	0.013	0.028	1.000
Panel D			Useful Factor Priced at Low Frequencies - Table 6a, Panel B and Table IA.6a, Panel C												
rank test	$\mathcal{J}_1$	**N = 1024**	0.089	0.138	0.191	0.676	1.000	1.000	0.191	0.147	0.124	0.095	0.071	0.137	0.110
	$\mathcal{J}_2$		0.045	0.076	0.105	0.504	1.000	1.000	0.105	0.081	0.070	0.044	0.031	0.067	0.044
	$\mathcal{J}_1$	**N = 256**	0.204	0.547	0.597	0.964	1.000	1.000	0.597	0.603	0.633	0.675	0.297	0.415	0.286
	$\mathcal{J}_2$		0.031	0.135	0.091	0.040	1.000	1.000	0.091	0.091	0.170	0.195	0.014	0.030	0.008
Panel E			Useful Factor Priced at Medium Frequencies - Table 6a, Panel C and Table IA.6a, Panel E												
rank test	$\mathcal{J}_1$	**N = 1024**	0.995	1.000	0.158	0.343	0.547	0.464	0.158	0.120	1.000	1.000	0.056	0.121	0.095
	$\mathcal{J}_2$		0.986	1.000	0.083	0.225	0.435	0.358	0.083	0.064	1.000	1.000	0.025	0.054	0.038
	$\mathcal{J}_1$	**N = 256**	0.988	1.000	0.597	0.984	0.996	0.794	0.597	0.578	1.000	1.000	0.290	0.412	0.277
	$\mathcal{J}_2$		0.834	1.000	0.082	0.100	0.183	0.226	0.082	0.083	1.000	1.000	0.013	0.032	0.009

Notes: This table reports rejection rates at a 5% significance level for the null hypothesis that the $N_A \times (k+1)$ matrix $X = \left(1_{N_A}, \beta\right)$ is of a reduced rank, i.e. $H_0 : \text{rank}\,(X) = k$. We report results for two rank tests: $\mathcal{J}_1$ from Gospodinov et al. (2017) that allows for conditional heteroskedasticity (see Equation IA.29) and $\mathcal{J}_2$ from Gospodinov and Robotti (2021) (assuming conditional homoskedasticity, see Equation IA.30). Our focus is on rank tests for a useless and a useful factor - for details on the simulations see Tables 4, 5, 6a and IA.4, IA.5 and IA.6a.