Internet Appendix for "Capital Share Risk in U.S. Asset Pricing: A Reappraisal"

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In Section I of this Internet Appendix, we present bootstrap versions of the employed rank and joint beta tests, and we investigate their empirical rejection rates via Monte Carlo simulations. In the same section, we describe the nonparametric bootstrap used in the paper. Section II contains some additional evidence for equity portfolio returns. Finally, we report results for nonequity asset classes in Section III. We use the same notation and figure format as in the paper.

I. Bootstrap Inference

We start by describing the bootstrap implementation of the rank and joint beta tests. Next, we explore the empirical rejection rates of the asymptotic and bootstrap versions of the rank and joint beta tests. Finally, we illustrate the nonparametric bootstrap method.

A. Bootstrap Rank and Joint Beta Tests

Recall that the identification condition for the second-pass risk premia is that the $N \times 2$ matrix $\mathbf{X} = [\mathbf{1}_N, \boldsymbol{\beta}_H]$ (for the case of one risk factor) is of full column rank. Let $\mathbf{I}_{N-1}$ be an $(N-1)\times(N-1)$ identity matrix and $\mathbf{P}$ denote an $N \times (N-1)$ orthonormal matrix ($\mathbf{P}'\mathbf{P} = \mathbf{I}_{N-1}$) whose columns are orthogonal to $\mathbf{1}_N$ such that

$$\mathbf{P}\mathbf{P}' = \mathbf{I}_N - \mathbf{1}_N(\mathbf{1}_N'\mathbf{1}_N)^{-1}\mathbf{1}_N'. \tag{1}$$

Using this notation, the null of reduced column rank, $H_0:\ \text{rank}(\mathbf{X}) = 1$, can be expressed as $H_0:\ \mathbf{P}'\boldsymbol{\beta}_H = \mathbf{0}_{N-1}$, where $\mathbf{0}_{N-1}$ is an $(N-1)$-vector of zeros. A simple Wald test of $H_0:\ \mathbf{P}'\boldsymbol{\beta}_H = \mathbf{0}_{N-1}$ can be performed using the following test statistic:

$$\mathcal{W}_T = (T-H)\hat{\boldsymbol{\beta}}_H'\mathbf{P}\hat{\mathbf{V}}^{-1}_{\mathbf{P}'\hat{\boldsymbol{\beta}}_H}\mathbf{P}'\hat{\boldsymbol{\beta}}_H, \tag{2}$$

where $\hat{\mathbf{V}}_{\mathbf{P}'\hat{\boldsymbol{\beta}}_H}$ is a consistent estimator of the long-run covariance matrix

$$\mathbf{V}_{\mathbf{P}'\hat{\boldsymbol{\beta}}_H} = \sum\nolimits_{j=-\infty}^{\infty} \mathbb{E}\left[\mathbf{m}_{t,H}\mathbf{m}'_{t+j,H}\right], \tag{3}$$

with $\mathbf{m}_{t,H} = \frac{(f_{t+H,t}-\mu_{f_H})}{\sigma^2_{f_H}}\mathbf{P}'\boldsymbol{\epsilon}_{t+H,t}$, $\mu_{f_H} = \mathbb{E}\left[f_{t+H,t}\right]$, and $\sigma^2_{f_H} = \mathrm{Var}\left[f_{t+H,t}\right]$. In the numerical implementation of the test, we use the Newey and West (1987) heteroskedasticity and autocorrelation consistent (HAC) estimator with a bandwidth set equal to H.

While under some regularity conditions the test $\mathcal{W}_T$ is asymptotically chi-squared distributed with $N-1$ degrees of freedom, this approximation will likely provide a very poor approximation of the finite-sample distribution for the reasons discussed in the paper: small T and large N and H (both relative to T) that further reduce the effective number of time series observations.[1] Before describing the bootstrap procedure for approximating the finite-sample distribution of the test $\mathcal{W}_T$, it is convenient to pre-multiply the first-pass regression model by $\mathbf{P}'$ and obtain the sample quantities that enter the test $\mathcal{W}_T$, which yields

$$\mathbf{P}'\mathbf{R}^e_{t+H,t} = \mathbf{P}'\boldsymbol{\alpha} + \mathbf{P}'\boldsymbol{\beta}_H f_{t+H,t} + \mathbf{P}'\boldsymbol{\epsilon}_{t+H,t}. \tag{4}$$

This model also facilitates imposing the null hypothesis of reduced rank $H_0: \mathbf{P}'\boldsymbol{\beta}_H = \mathbf{0}_{N-1}$ in the bootstrap sample. Under the null, we have

$$\mathbf{P}'\mathbf{R}^e_{t+H,t} = \boldsymbol{\mu}_{\mathbf{P}'\mathbf{R}^e} + \mathbf{P}'\boldsymbol{\epsilon}_{t+H,t}, \tag{5}$$

where $\boldsymbol{\mu}_{\mathbf{P}'\mathbf{R}^e} = \mathbb{E}\left[\mathbf{P}'\mathbf{R}^e_{t+H,t}\right]$. Let $\mathbf{P}'\hat{\boldsymbol{\epsilon}}_{t+H,t}$ denote the OLS estimate of $\mathbf{P}'\boldsymbol{\epsilon}_{t+H,t}$ and $\hat{\boldsymbol{\mu}}_{\mathbf{P}'\mathbf{R}^e}$ be the sample estimate of $\boldsymbol{\mu}_{\mathbf{P}'\mathbf{R}^e}$. Stack the H-period factor $f_{t+H,t}$ and the $(N-1)$-vector $\tilde{\mathbf{R}}^e_{t+H,t} = \hat{\boldsymbol{\mu}}_{\mathbf{P}'\mathbf{R}^e} + \mathbf{P}'\hat{\boldsymbol{\epsilon}}_{t+H,t}$ in a $(T-H)\times N$ matrix $\mathbf{Z}$ with rows $\mathbf{z}_t = [f_{t+H,t}, (\tilde{\mathbf{R}}^e_{t+H,t})']$ for $t = 1, \ldots, T-H$. The bootstrap samples are constructed by drawing with replacement blocks of l $(1 \leq l < T-H)$ observations from matrix $\mathbf{Z}$, denoted by $\mathbf{Z}^* = \{(\mathbf{z}^*_1, \mathbf{z}^*_2, \ldots, \mathbf{z}^*_l), (\mathbf{z}^*_{l+1}, \mathbf{z}^*_{l+2}, \ldots, \mathbf{z}^*_{2l}), \ldots, (\mathbf{z}^*_{T-l-H}, \mathbf{z}^*_{T-l+1-H}, \ldots, \mathbf{z}^*_{T-H})\}$ with $\mathbf{z}^*_t = [f^*_{t+H,t}, (\tilde{\mathbf{R}}^{e*}_{t+H,t})']$ being the resampled analog of the original data $\mathbf{z}_t = [f_{t+H,t}, (\tilde{\mathbf{R}}^e_{t+H,t})']$. Using the bootstrap sample, we obtain the estimated quantities $\mathbf{P}'\hat{\boldsymbol{\beta}}^*_H$ and $\mathbf{P}'\hat{\boldsymbol{\epsilon}}^*_{t+H,t}$ by running an OLS regression of $\tilde{\mathbf{R}}^{e*}_{t+H,t}$ on $f^*_{t+H,t}$ (and a constant). Then,

[1]The H-period overlapping data also induces strong serial correlation of a telescoping sum pattern. It is widely documented that the Newey and West (1987) HAC estimator is not well-suited to capture this type of serial dependence. We also experimented with the Hansen and Hodrick (1980) HAC estimator, but this estimator is not guaranteed to be positive semi-definite. This is the case in LLM's empirical application given the large N and the relatively small T.

the bootstrap analog of $\mathcal{W}_T$ for the j-th bootstrap sample is constructed as

$$\mathcal{W}^*_{T,j} = (T-H)\hat{\boldsymbol{\beta}}^{*\prime}_H \mathbf{P}\hat{\mathbf{V}}^{*-1}_{\mathbf{P}'\hat{\boldsymbol{\beta}}^*_H}\mathbf{P}'\hat{\boldsymbol{\beta}}^*_H, \tag{6}$$

where $\hat{\mathbf{V}}^*_{\mathbf{P}'\hat{\boldsymbol{\beta}}^*_H}$ denotes the HAC estimator of $\mathbf{V}_{\mathbf{P}'\hat{\boldsymbol{\beta}}_H}$, with the bootstrap sample analog of $\mathbf{m}_{t,H}$ being $\hat{\mathbf{m}}^*_{t,H} = \frac{(f^*_{t+H,t}-\hat{\mu}^*_{f^*_H})}{\hat{\sigma}^{*2}_{f^*_H}}\mathbf{P}'\hat{\boldsymbol{\epsilon}}^*_{t+H,t}$, $\hat{\mu}^*_{f^*_H} = \frac{1}{T-H}\sum_{t=1}^{T-H} f^*_{t+H,t}$, and $\hat{\sigma}^{*2}_{f^*_H} = \frac{1}{T-H}\sum_{t=1}^{T-H}(f^*_{t+H,t} - \hat{\mu}^*_{f^*_H})^2$. With B bootstrap replications, the bootstrap p-value of the rank test is computed as $\frac{1}{B}\sum_{j=1}^{B}\mathbb{I}\left\{\mathcal{W}^*_{T,j} > \mathcal{W}_T\right\}$, where $\mathbb{I}\{\cdot\}$ is the indicator function.

The bootstrap test of the null $H_0: \boldsymbol{\beta}_H = \mathbf{0}_N$ is constructed similarly but without pre-multiplying by the matrix $\mathbf{P}'$. While the two tests yield almost identical results for models with a single spurious factor, differences emerge in the presence of useful factors in single- or multi-factor models. For example, since the identification condition is concerned with the matrix $\mathbf{X} = [\mathbf{1}_N, \boldsymbol{\beta}_H]$, the rank of $\mathbf{X}$ can be compromised if $\boldsymbol{\beta}_H = \mathbf{c}$ for some $\mathbf{c} \neq \mathbf{0}_N$. Furthermore, in multi-factor models, $\boldsymbol{\beta}_H$ is a matrix and rank failure can also occur if two or more of its columns are linear combinations of each other (even if, individually, they are different than a zero vector, that is, the factors are not spurious). The bootstrap rank test described above can accommodate these possibilities with the added advantage, as we show in the next subsection, of good size control when the number of effective time series observations is small.

B. Size Properties of the Joint Beta and Rank Tests

To evaluate the size properties of the asymptotic and bootstrap versions of the joint beta and rank tests, we set up a Monte Carlo experiment where we generate a spurious factor, that is, a factor that is independent of the test asset returns. In order to accomplish this and preserve the salient features of the data, we start by approximating the capital share dynamics by the autoregressive (AR) of order one, AR(1), process

$$KS_{t+1} = \delta + \rho KS_t + \varepsilon_{t+1}, \tag{7}$$

where $\rho < 1$ and ε_{t+1} is a mean-zero error term with variance σ^2. (Other approximations could also be employed.) Let $\hat{\sigma}^2$ denote the sample estimate of σ^2 from the actual data. Then, we generate ε°_{t+1} as $N(0, \hat{\sigma}^2)$ independently from $[R_{1,t+1}, \ldots, R_{N,t+1}]'$, and we construct a simulated capital share series $KS^\circ_{t+1} = \hat{\delta} + \hat{\rho}KS^\circ_t + \varepsilon^\circ_{t+1}$ for some initial value KS_1, where $\hat{\delta}$ and $\hat{\rho}$ denote the OLS estimates (from the actual data) of δ and ρ, respectively. Subsequently, we use the simulated capital share process to construct the H-period factor $f^{\circ}_{t+H,t} = KS^{\circ}_{t+H}/KS^{\circ}_{t}$. Similarly, we generate one-period returns as multivariate normally distributed with mean and covariance matrix estimated from the actual data. The H-horizon return compounding is then performed as usual by taking the moving product over a sliding window of length H. For the bootstrap versions of the two tests, we impose the null of rank deficiency on the compounded returns (as explained in Section I.A), and we use the block bootstrap to compute the rank and joint beta test statistics.[2]

In the Monte Carlo experiment, we consider two sample sizes: $T = 202$, the (before transformations) sample size in LLM, and $T = 1,000$, a sufficiently large sample size to determine whether the empirical size of the various tests improves as T increases. The number of Monte Carlo runs is set equal to 10,000. The chosen horizons are $H = 1$, 4, and 8, and the number of bootstrap replications for the bootstrap versions of the joint beta and rank tests is set equal to $B = 399$. Finally, the test portfolios are the 10 long-run reversal portfolios ($N = 10$) and the 25 size and book-to-market sorted portfolios ($N = 25$), respectively.

Table IA.I reports our simulations results, where Panels A and B are for the asymptotic versions of the tests while Panels C and D are for their bootstrap counterparts.

Table IA.I about here

The results in Panel B are striking. For $N = 25$, $T = 202$, and $H = 4, 8$, the asymptotic implementation of the joint beta and rank tests leads to empirical rejection rates close to 100% at a 5% nominal level of the tests. Even for $T = 1,000$, these rejection rates exceed 50% and 78% for $H = 4$ and $H = 8$, respectively. Certainly, a smaller N and a larger T help, but the overrejections of the asymptotic versions of these tests are still substantial, as emphasized in Panel A for the 10 long-run reversal portfolios. Panels C and D display a dramatic size improvement when considering the bootstrap implementation of the joint beta and rank tests. The size properties of the tests are now very good for $N = 10$, regardless of the chosen overlapping horizon H. For $N = 25$, the tests slightly underreject for $T = 202$, but their empirical size approaches the nominal level of the tests as T increases.[3] To our knowledge, we are the first to document these huge size distortions for

[2] We employ a block size $M = H$. Moreover, we use the Newey and West (1987) HAC estimator with a bandwidth set equal to H in the computation of the asymptotic and bootstrap versions of the tests. It should be noted though that in general the Newey and West (1987) HAC estimator is not very well-suited to capture the type of persistence arising from overlapping data.

[3] We attribute the slight size distortions of the bootstrap to our choice of block size, $M = H$. A more judicious or

the asymptotic joint beta and rank tests in an overlapping setting. In addition, this is the first study to document the impressive size corrections that can be obtained by using the bootstrap versions of the joint beta and rank tests. In summary, this simulation evidence suggests that the bootstrap-based identification tests used in the paper should be fairly reliable for the sample sizes and compounding horizons considered by LLM.

C. Nonparametric Bootstrap-Based Confidence Intervals

In this subsection, we describe a nonparametric bootstrap method that is agnostic to the underlying data generating mechanism but flexible enough to account for the salient features of the data. Besides being robust to possible model misspecification, it also does not rely on a parametric structure that could be poorly identified due to the presence of spurious or nearly spurious factors.

The version of the method that we present here resamples the one-period factor and returns and then constructs the H-period series. We assume that the capital share dynamics can be approximated as in Eq. (7). As mentioned in the paper, the estimate of the risk premium on the capital share factor, $\hat{\lambda}_H$, is unchanged if the estimation is performed with the gross returns $\mathbf{R}$ instead of the excess returns $\mathbf{R}^e$. Stack the OLS residuals $\hat{\varepsilon}_{t+1}$ (obtained from Eq. (7)) and the gross returns on the N assets in the matrix

$$\mathbf{Z} = \begin{bmatrix} \hat{\varepsilon}_2 & R_{1,2} & \dots & \dots & R_{N,2} \\ \dots & \dots & & & \dots \\ \hat{\varepsilon}_{t+1} & R_{1,t+1} & \dots & \dots & R_{N,t+1} \\ \dots & \dots & & & \dots \\ \hat{\varepsilon}_T & R_{1,T} & \dots & \dots & R_{N,T} \end{bmatrix}. \tag{8}$$

We resample this matrix by block bootstrap, with block size M, to accommodate any serial correlation in the AR residuals and one-period returns as well as possible conditional heteroskedasticity. It is important to remark that this fully preserves the cross-sectional covariance structure of the data. Let $\mathbf{Z}^*$ denote the resampled $\mathbf{Z}$ matrix with a typical row $[\hat{\varepsilon}^*_{t+1}, R^*_{1,t+1}, \dots, R^*_{N,t+1}]$, which, because of the block bootstrap structure for $M > 1$, retains the dependence with its adjacent rows. The bootstrap sample for the capital share series is then obtained as $KS^*_{t+1} = \hat{\delta} + \hat{\rho} KS^*_t + \hat{\varepsilon}^*_{t+1}$ for some initial value KS_1,[4] and the H-period bootstrap factor is constructed as $f^*_{t+H,t} = KS^*_{t+H}/KS^*_t$. The bootstrap returns at $H > 1$ are constructed by compounding one-period bootstrap returns as

data-driven selection of M would likely eliminate these distortions.

[4]To start the recursion, we draw randomly an observation from the sample. We also experimented with the first observation in the sample and found the difference in results to be negligible.

$R^*_{j,t+2,t} = R^*_{j,t+1}R^*_{j,t+2}, \ldots, R^*_{j,t+H,t} = \prod_{h=1}^{H} R^*_{j,t+h}$ for $j = 1, \ldots, N$. The bootstrap estimates of $\boldsymbol{\beta}_H$, $\boldsymbol{\beta}^*_H$, are obtained from an OLS time-series regression of $\mathbf{R}^*_{t+H,t}$ on $f^*_{t+H,t}$ and a constant, while the risk premium bootstrap estimate, λ^*_H, is computed from an OLS cross-sectional regression of $\boldsymbol{\mu}^*_R$, the vector of sample means for $\mathbf{R}^*_{t+1} = [R^*_{1,t+1}, \ldots, R^*_{N,t+1}]'$, on $[\mathbf{1}_N,\ \boldsymbol{\beta}^*_H]$. For a significance level α, the $100(1-\alpha)\%$ bootstrap confidence intervals are given by $[q^*(\alpha/2), q^*(1-\alpha/2)]$, where $q^*(\eta)$ denotes the η-th quantile of the empirical distribution of λ^*_H.

II. Additional Results for Equities

In what follows, we consider alternative factors to shed more light on the effects of the H-period overlapping on pricing and statistical inference. We first consider the market factor (the excess return on the value-weighted NYSE-AMEX-NASDAQ stock market index) that provides a natural benchmark for one-period nonoverlapping returns. We then assess how the statistical properties of its risk premium are affected by overlapping (compounding) the market return over H periods. We also conduct a similar analysis for LLM's consumption factor, measured as expenditures on nondurables and services (excluding shoes and clothing). The frequency and sample period are the same as the ones considered in the paper.

In what follows, the H-period market excess return is defined as $R^e_{m,t+H,t} = \prod_{h=1}^{H} R_{m,t+h} - \prod_{h=1}^{H} R_{f,t+h}$, which is identical to the way the H-period excess returns on the test assets are constructed. We start with the observation, for which we provide statistical evidence below, that the beta estimates for the market factor are significantly different from zero. Figure IA.1 plots the 95% bootstrap confidence intervals (based on LLM's bootstrap procedure) for the market risk premium.

Figure IA.1 about here

In sharp contrast with Figure 1 for the capital share factor in the paper, the confidence intervals for the market risk premium are much wider and do not exhibit the pronounced tightening with H that we observe for the KS factor.

Next, we modify slightly LLM's bootstrap to address some issues in their resampling procedure. First, in LLM's block bootstrap implementation in the first-pass, there seems to be an error that results in bootstrap data that exhibits much less persistence than the actual data (for $H > 1$). This is related not to the choice of block size but to the block resampling itself. Second, in constructing bootstrap average returns for the second-pass, LLM resample the cross-sectional average pricing errors. Since these average pricing errors are small, this induces very little variation in the bootstrap average returns (which explains the $[-0.00, 0.00]$ confidence intervals reported in LLM's Internet Appendix). Furthermore, this resampling is done independently of the resampling in the first-pass. Since the underlying one-period returns in both stages are the same, this creates some logical inconsistency in the two sets of bootstrap returns. We modify LLM's bootstrap method to fix these two issues. For the second-pass regression, we resample the panel ($(T - H) \times N$ matrix) of pricing errors along the time series dimension and then compute the cross-sectional averages to construct bootstrap average pricing errors. Also, to ensure internal consistency, we implement the block bootstrap on the $(T - H) \times (1 + 2N)$ stacked matrix of the AR(1) factor residuals, N time series of first-pass regression residuals, and N time series of pricing errors. To be in line with LLM's recommendation, we use a block size $M = 3$ for all H, although, to preserve the serial correlation patterns induced by overlapping, the block size should be a function of the overlapping horizon and it should be at least as large as H (results for this choice of M are available upon request). The block bootstrap samples are then constructed using the circular block bootstrap of Politis and Romano (1994).[5] The bootstrap series for the relevant variables are obtained using the parametric structure of the AR(1) model for the factor as well as the two-pass model of returns.

Figure IA.2 presents the results of this modified bootstrap for the market risk premium.

Figure IA.2 about here

Relative to Figure IA.1, the confidence intervals in Figure IA.2 are wider and the market risk premium estimate is statistically insignificant (at the 5% significance level) at all horizons H. These findings are largely consistent with the insignificant results for the market risk premium that are typically obtained based on one-period returns. Importantly, we do not observe the pronounced pattern of shrinking confidence intervals that occurs for the capital share factor. Therefore, the question is: what is the reason for these vastly different results?

To gain some further intuition, in Figure IA.3 we present the bootstrap p-values of the joint

[5] The circular block bootstrap of Politis and Romano (1994) is also used in the nonparametric method in Section I.C.

beta and rank tests for the market factor.

Figure IA.3 about here

As expected, the joint beta test always strongly rejects the null $H_0: \boldsymbol{\beta}_H = \mathbf{0}_N$ at all horizons. The rank test also rejects the null of reduced rank at short horizons but not at long overlapping horizons. Thus, even for the market factor, LLM's bootstrap (or any other statistical inference procedure that maintains the identification assumption) would not be valid for intermediate or large H, as it would tend to underestimate the true uncertainty. The different outcomes between the joint beta and rank tests for large H also nicely illustrate the more general nature of the rank test outlined in Section I above. Two other interesting issues emerge. First, since the sampling variation tends to increase with H, the rank test cannot reliably differentiate $\hat{\boldsymbol{\beta}}_H$ from a vector of ones. This is a manifestation of the reduction of the number of effective time series observations induced by overlapping that we mention in the paper. Second, the rank test cannot reject the null of a reduced rank for "All Equities." In this case, the number of assets is very large ($N = 85$) relative to the time series sample size and the number of effective time series observations per test asset (moment condition) is small. Given the lack of sufficient sample information, it is not surprising that based on the rank test we cannot reject the null of lack of identification. This stands in sharp contrast with the asymptotic tests (not reported here) that provide a very poor approximation in this setting.

As an additional robustness check of the properties of the bootstrap confidence intervals of LLM's method when the factor is potentially spurious, we consider the H-period growth of non-durable consumption in LLM's dataset. As for the capital share factor, based on the joint beta and rank tests for the consumption factor in Figure IA.4, we cannot reject the null of identification failure.

Figure IA.4 about here

In addition, Figure IA.5 plots LLM's 95% bootstrap confidence intervals for the consumption risk premium.

Figure IA.5 about here

To highlight the similarities with the capital share factor, the vertical axis is the same as in Figure 1 in the paper. The tightness of the confidence intervals is again remarkable even though $\hat{\lambda}_H$ is statistically significant only for three of the five sets of test assets.

In summary, this additional evidence strongly suggests that the likely source of the extremely tight confidence intervals for LLM's capital share factor is the identification failure. (It is important to remind the readers that the parametric bootstrap proposed by LLM maintains the assumption that the model is fully identified, that is, $\mathbf{X}$ is of full column rank.) Overlapping data appears to amplify and obscure (to some standard diagnostic checks) this identification failure as the persistence induced by overlapping introduces a new dimension for spurious relationships. Given the small number of effective time series observations (due to large N and H), traditional asymptotic methods may lead to highly misleading inference. Overall, the interaction between identification failure and high persistence of the variables in the first-pass appears to be the main driver of the counterintuitive tightness of the confidence intervals.

III. Other Asset Classes

LLM claim that their capital share factor is also priced in the cross-section of expected returns on nonequity portfolios such as corporate bonds ("Bonds"), sovereign bonds ("Sovereign Bonds"), index options ("Options"), and credit default swaps ("CDS"). The data details and the corresponding sample periods can be found in LLM's Section I. For all these asset classes, the time series sample size is substantially smaller than the one for equity portfolios. For the case of CDS, for example, the number of time series observations, before overlapping, is $T = 47$, whereas the number of test assets is $N = 20$. This clearly represents an extreme scenario where drawing reliable inferences could be challenging for any econometric method. Figure IA.6 parallels Figure 1 in the paper.

Figure IA.6 about here

Based on LLM's bootstrap method, the capital share factor is always priced regardless of the chosen horizon H. (The only exception is for sovereign bonds at $H = 10$.) Different from equity portfolios, the evidence of pricing for the capital share factor is very strong at $H = 1$, the one-period (nonoverlapping) scenario. The figure also displays an overall confidence interval tightening pattern for these additional asset classes, which is consistent with the one in Figure 1 in the paper. However, Figure IA.7 shows that this strong evidence of pricing is the likely artifact of identification failure.

Figure IA.7 about here

For these alternative asset classes, factor spuriousness seems to be as strong and pervasive as for equities. Moreover, the p-values of the beta and rank tests are generally close to each other at the various horizons H.

Similar to Figure 5 in the paper, Figure IA.8 plots the sample and the simulated R^2 profiles for $H = 1, \ldots, 16$.

Figure IA.8 about here

The figure reveals that the sample pattern of high cross-sectional R^2 values is consistent with a spurious factor that is independent of the test asset returns. The "commonality" between the factor and the returns can be traced back to the common persistent pattern from H-period overlapping, amplified by the even smaller effective time series sample size relative to the equity portfolio case.

Finally, Figure IA.9 plots the 95% confidence intervals based on the nonparametric bootstrap for the capital share risk premium at various horizons H.

Figure IA.9 about here

The evidence seems to always support absence of pricing, regardless of the asset class and chosen compounding horizon. In summary, all this empirical and simulation evidence points to spuriousness and lack of genuine pricing for the capital share factor proposed by LLM.

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Table IA.I
Size Properties of the Joint Beta and Rank Tests

The table presents Monte Carlo simulation results for the empirical size of the asymptotic and bootstrap versions of the joint beta and rank tests. We consider two sample sizes ($T = 202$, the length of LLM's original sample, and $T = 1,000$) and three compounding horizons ($H = 1$, 4, and 8). The number of simulated paths for the returns and factor is 10,000. For each of the 10,000 Monte Carlo iterations, the p-values for the bootstrap versions of the tests are computed based on 399 replications. The test portfolios are the 10 long-run reversal portfolios ($N = 10$) and the 25 size and book-to-market sorted portfolios ($N = 25$), respectively. The factor is calibrated to the dynamics of LLM's KS series and is generated independently of the test asset returns. (See Section I.B.)

Panel A: Asymptotic Tests ($N = 10$)							
		Joint Beta Test			Rank Test		
	T	10%	5%	1%	10%	5%	1%
$H = 1$	202	0.234	0.149	0.056	0.218	0.135	0.048
	1,000	0.122	0.067	0.013	0.119	0.062	0.013
$H = 4$	202	0.657	0.566	0.397	0.606	0.508	0.337
	1,000	0.311	0.213	0.088	0.291	0.197	0.080
$H = 8$	202	0.888	0.846	0.748	0.842	0.787	0.668
	1,000	0.438	0.330	0.164	0.408	0.297	0.143
Panel B: Asymptotic Tests ($N = 25$)							
		Joint Beta Test			Rank Test		
	T	10%	5%	1%	10%	5%	1%
$H = 1$	202	0.630	0.528	0.340	0.604	0.496	0.309
	1,000	0.190	0.116	0.034	0.187	0.109	0.032
$H = 4$	202	0.996	0.992	0.981	0.993	0.988	0.974
	1,000	0.640	0.536	0.339	0.623	0.514	0.315
$H = 8$	202	1.000	1.000	1.000	1.000	1.000	1.000
	1,000	0.869	0.804	0.655	0.850	0.782	0.619
Panel C: Bootstrap Tests ($N = 10$)							
		Joint Beta Test			Rank Test		
	T	10%	5%	1%	10%	5%	1%
$H = 1$	202	0.089	0.042	0.007	0.091	0.043	0.007
	1,000	0.099	0.048	0.008	0.095	0.049	0.008
$H = 4$	202	0.120	0.053	0.009	0.123	0.056	0.008
	1,000	0.130	0.071	0.015	0.129	0.069	0.016
$H = 8$	202	0.084	0.030	0.003	0.085	0.035	0.003
	1,000	0.127	0.064	0.011	0.126	0.066	0.011
Panel D: Bootstrap Tests ($N = 25$)							
		Joint Beta Test			Rank Test		
	T	10%	5%	1%	10%	5%	1%
$H = 1$	202	0.055	0.022	0.002	0.059	0.023	0.002
	1,000	0.100	0.049	0.008	0.099	0.047	0.008
$H = 4$	202	0.062	0.024	0.001	0.066	0.024	0.002
	1,000	0.145	0.073	0.015	0.144	0.075	0.015
$H = 8$	202	0.027	0.006	0.000	0.026	0.006	0.000
	1,000	0.099	0.043	0.005	0.100	0.044	0.005

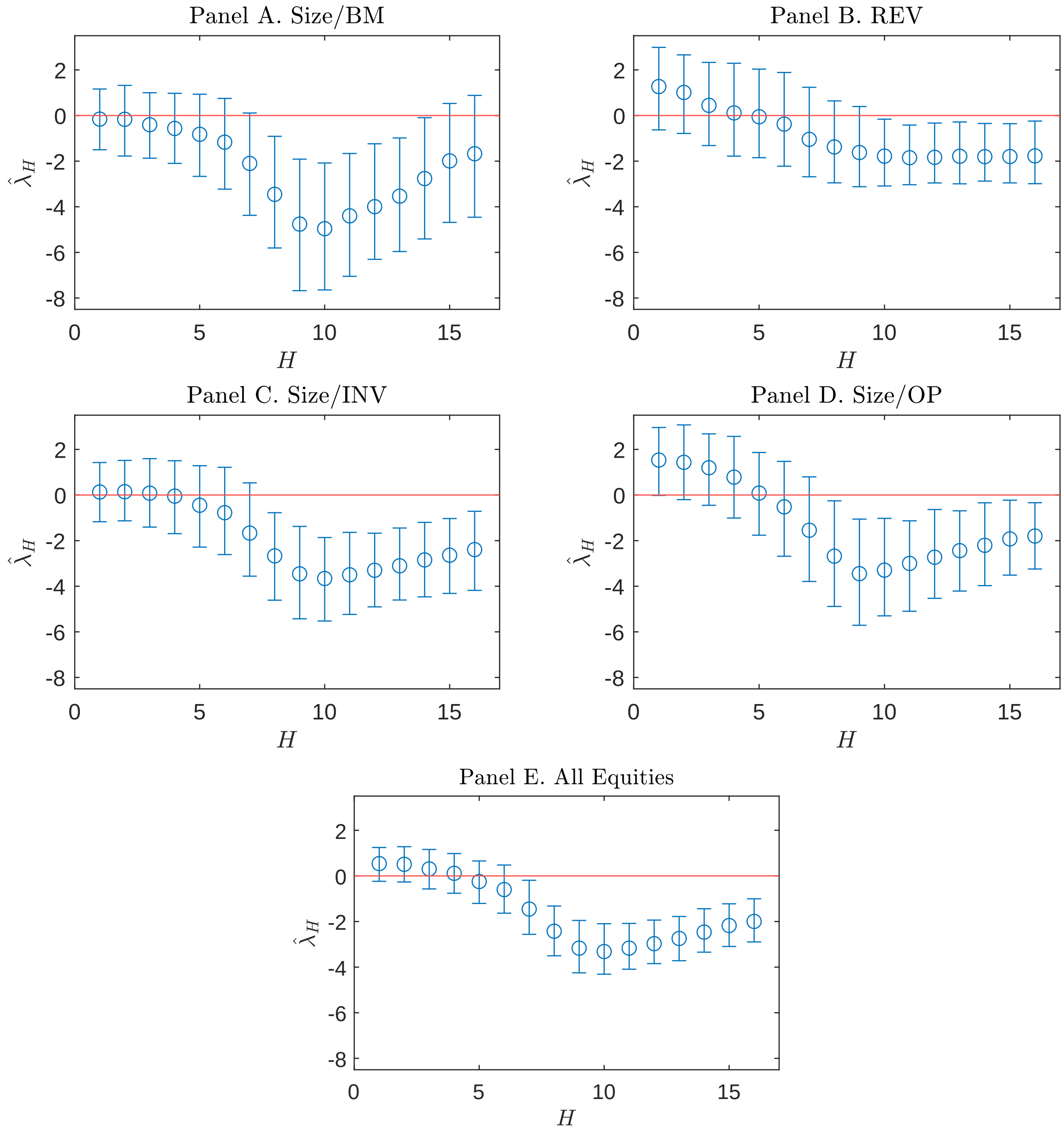

Figure IA.1. Expected return-beta regressions with the market factor (LLM's bootstrap). The plots display the market risk premium estimate, $\hat{\lambda}_H$, and its 95% bootstrap confidence interval for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and market exposure are measured. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM for the equity portfolio case.

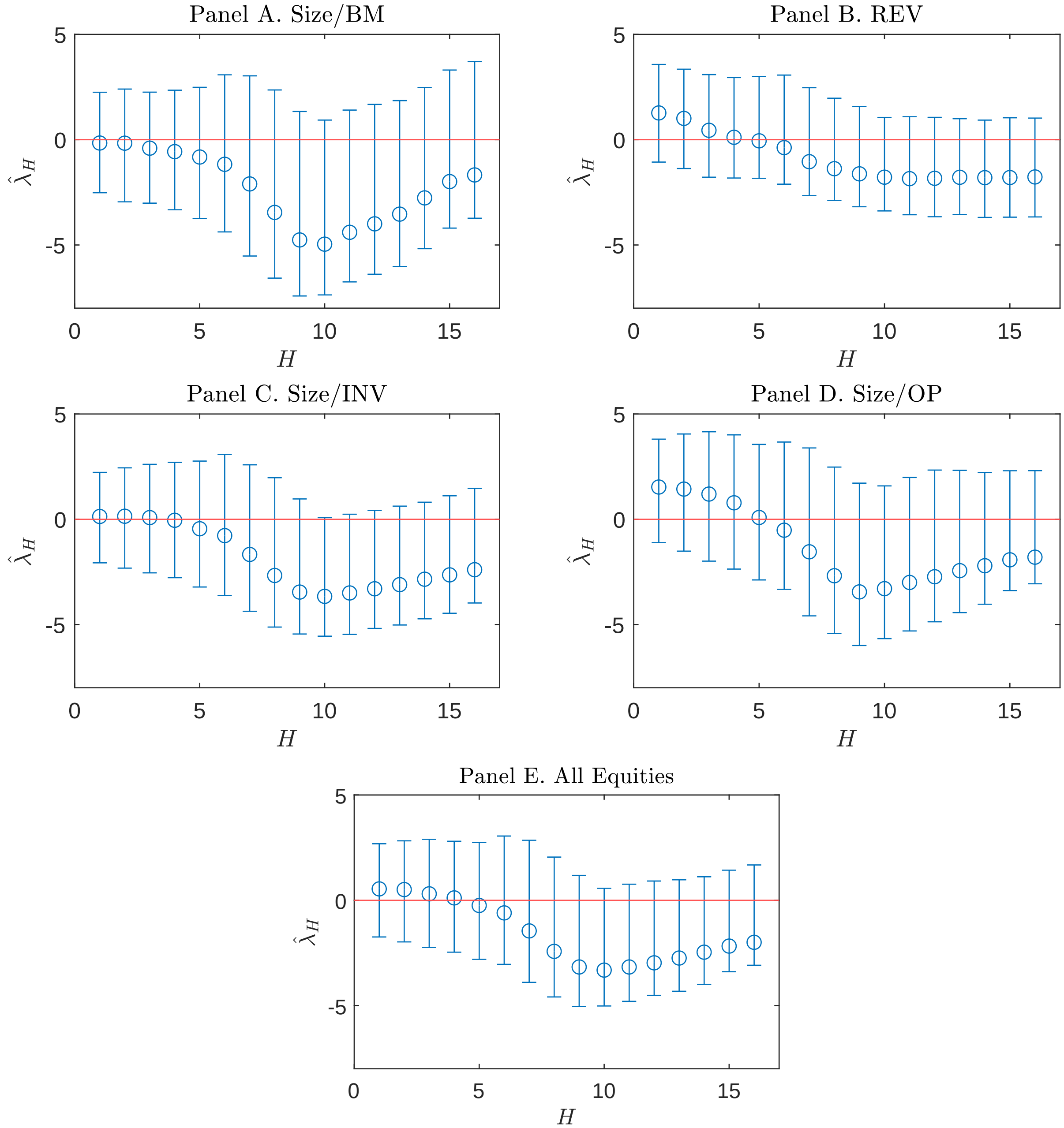

Figure IA.2. Expected return-beta regressions with the market factor (modified LLM's bootstrap). The plots display the market risk premium estimate, $\hat{\lambda}_H$, and its 95% bootstrap confidence interval for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and market exposure are measured. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM for the equity portfolio case.

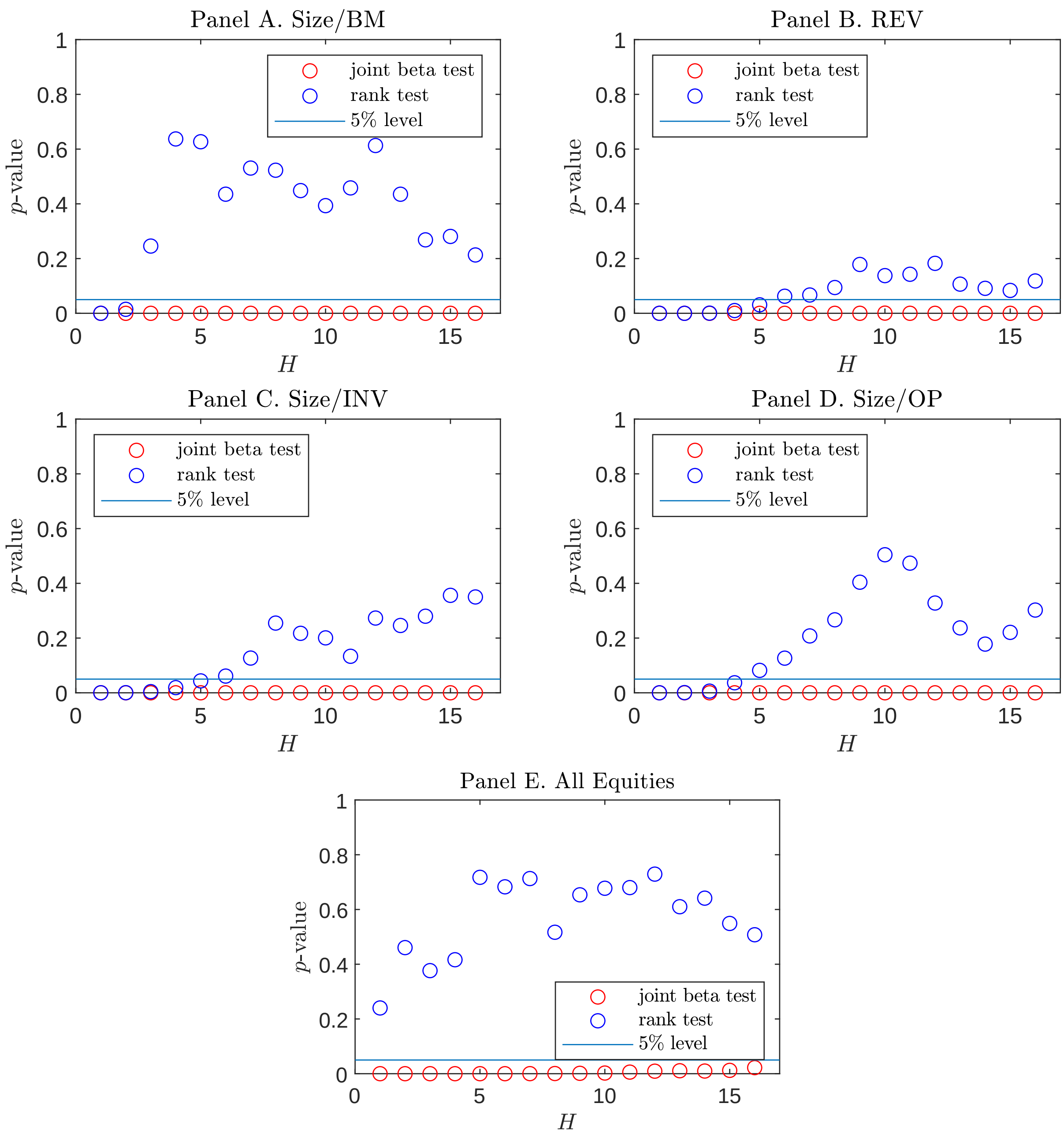

Figure IA.3. Joint beta and rank tests with the market factor. The plots display the p-values of the bootstrap joint beta (red circles) and rank (blue circles) tests of model identification for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and market exposure are measured. The horizontal blue line represents the 5% nominal size of the tests. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM for the equity portfolio case.

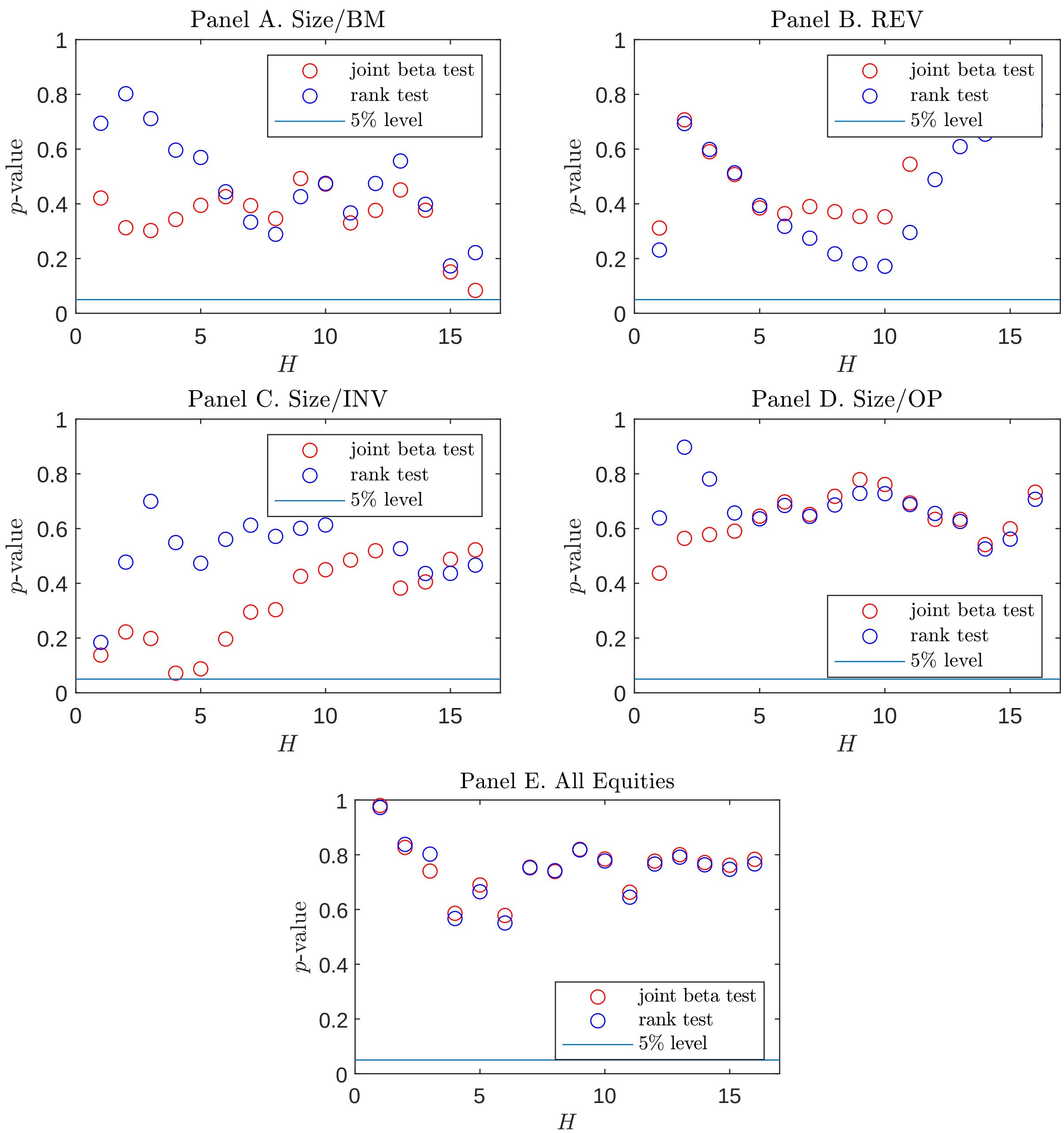

Figure IA.4. Joint beta and rank tests with the consumption factor. The plots display the p-values of the bootstrap joint beta (red circles) and rank (blue circles) tests of model identification for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and consumption exposure are measured. The horizontal blue line represents the 5% nominal size of the tests. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM for the equity portfolio case.

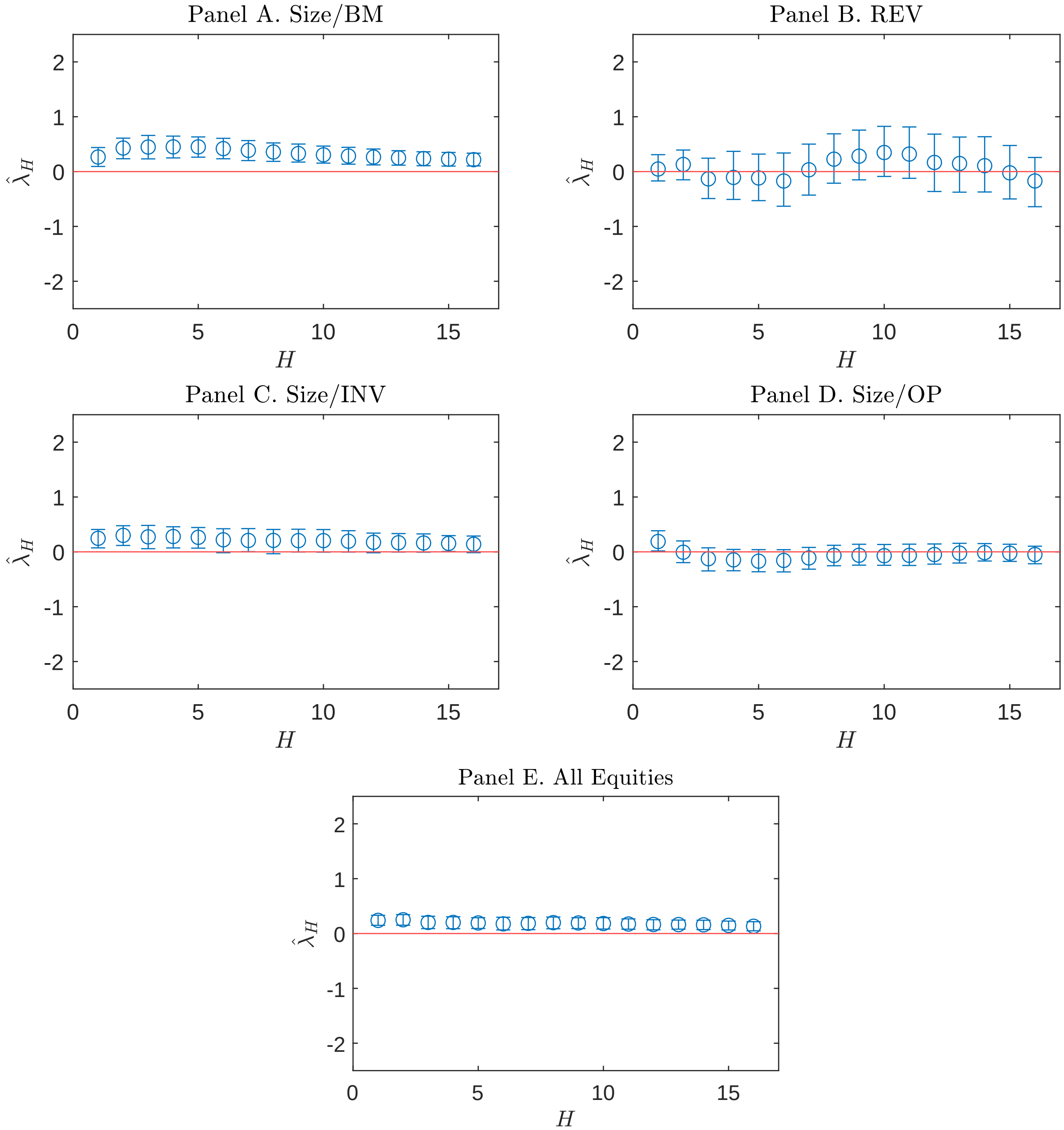

Figure IA.5. Expected return-beta regressions with the consumption factor (LLM's bootstrap). The plots display the consumption risk premium estimate, $\hat{\lambda}_H$, and its 95% bootstrap confidence interval for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and consumption exposure are measured. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM for the equity portfolio case.

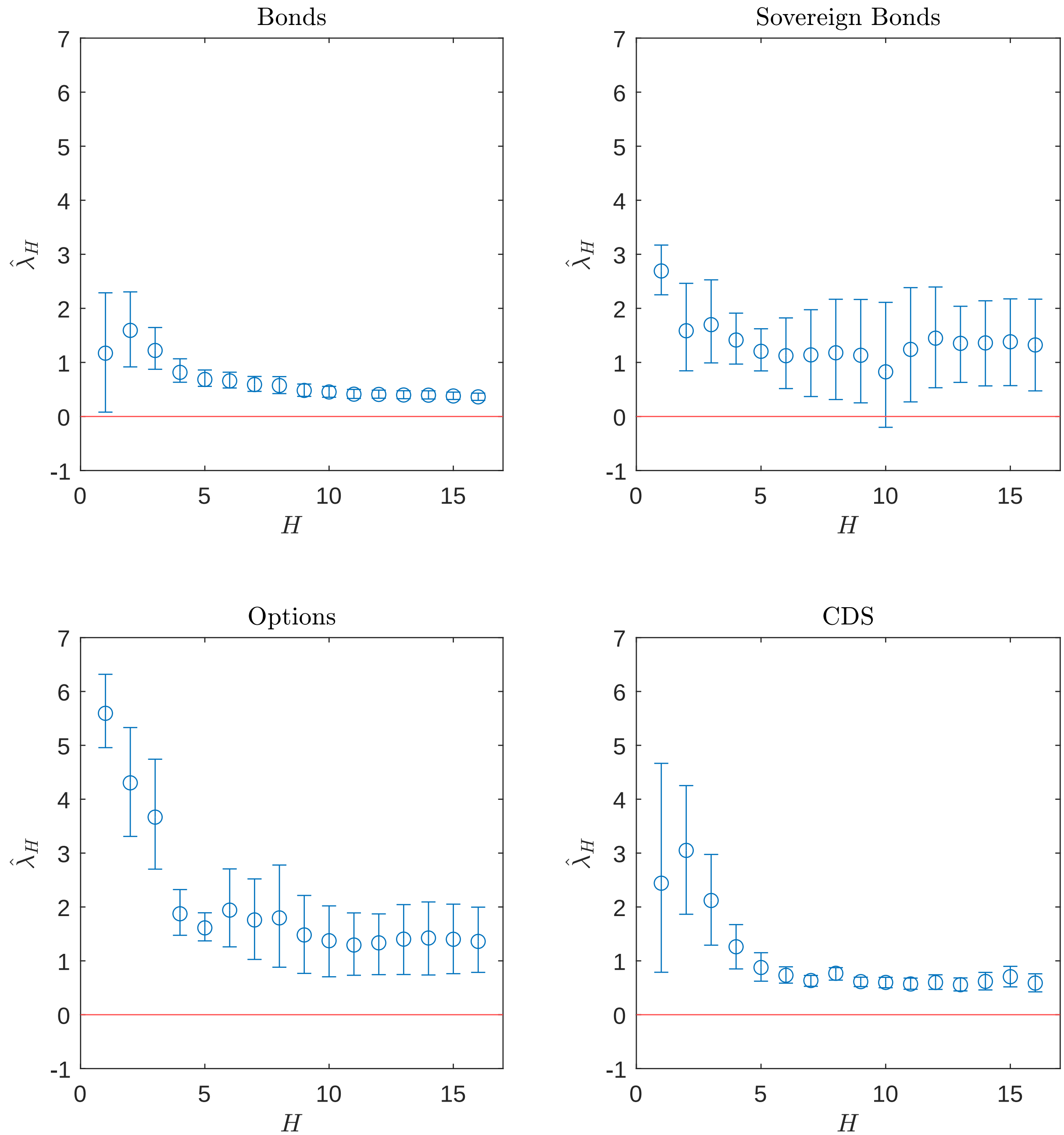

Figure IA.6. Expected return-beta regressions for nonequity asset classes (LLM's bootstrap). The plots display LLM's capital share risk premium estimate, $\hat{\lambda}_H$, and its 95% bootstrap confidence interval for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and capital share exposure are measured. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM. (See their Section I.)

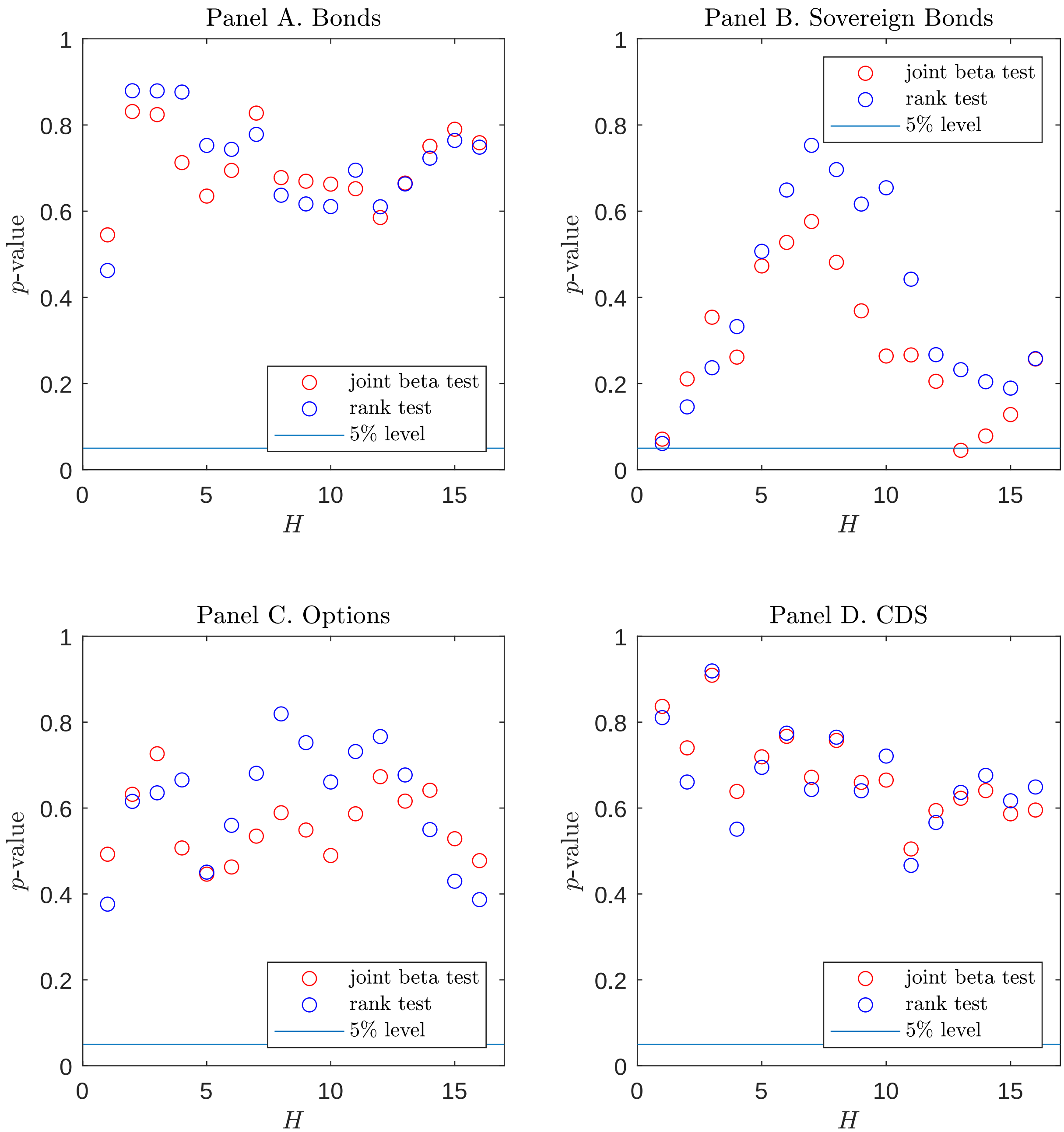

Figure IA.7. Joint beta and rank tests for nonequity asset classes. The plots display the p-values of the bootstrap joint beta (red circles) and rank (blue circles) tests of model identification for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and capital share exposure are measured. The horizontal blue line represents the 5% nominal size of the tests. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM. (See their Section I.)

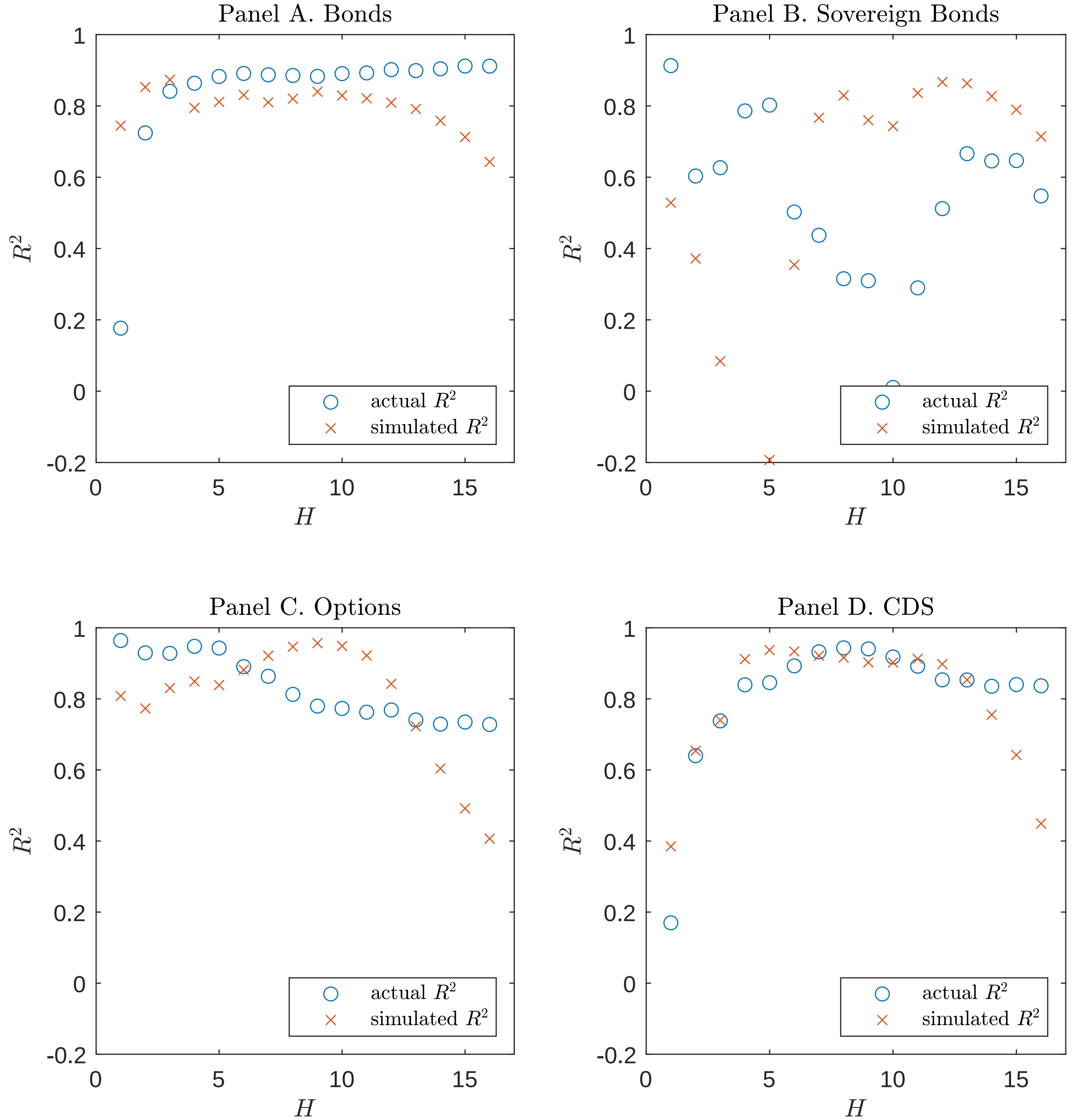

Figure IA.8. R^2 profiles from actual and simulated data for nonequity asset classes. The plots display the cross-sectional R^2 from the actual data and from simulated sample paths of a spurious factor for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and capital share exposure are measured. The number of simulated factor paths is 10,000. The test returns are kept fixed at their observed values across the simulations. The sample period is the one considered by LLM. (See their Section I.)

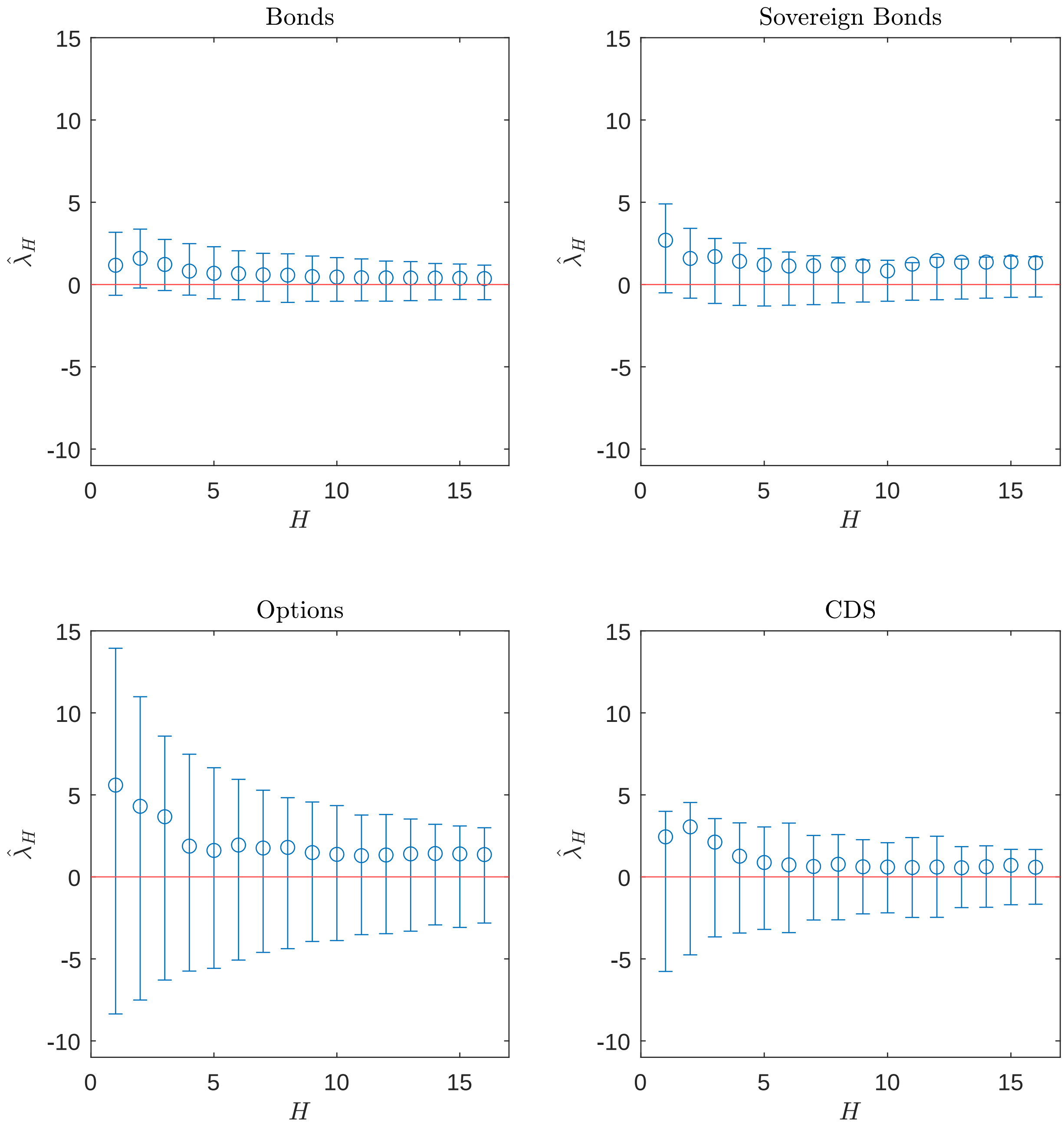

Figure IA.9. Expected return-beta regressions for nonequity asset classes (nonparametric bootstrap). The plots display LLM's capital share risk premium estimate, $\hat{\lambda}_H$, and its 95% bootstrap confidence interval for $H = 1, \ldots, 16$. H indicates the horizon in quarters over which returns and capital share exposure are measured. The number of bootstrap replications is 10,000. The sample period is the one considered by LLM. (See their Section I.)