Additional Empirical Results for

Misspecification-Robust Inference in Linear Asset-Pricing Models with Irrelevant Risk Factors

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In this appendix, we provide some additional empirical results that are not included in the paper.

A Carry Trade Portfolios and Consumption Growth

Lustig and Verdelhan (2007) claim that aggregate consumption growth risk explains the excess returns to borrowing U.S. dollars to finance lending in other currencies. In this section, we revisit their findings in light of our new theoretical and simulation results.

The return data are the annual excess returns on eight currency portfolios from 1953 until 2002.[1] It is well-known that when only excess returns are used in the analysis, the mean of the SDF cannot be identified. As a result, researchers have to choose some normalization of the SDF. We follow Kan and Robotti (2008) and employ the modified HJ-distance as our misspecification measure. Kan and Robotti (2008) show that the modified HJ-distance measure has the desirable property of being invariant to affine transformations of the factors. Under this metric and when the model is linear, the SDF is written as

$$y_t(\gamma) = 1 - \gamma'(f_t - E[f_t]).$$

It should be noted that the theory developed in the paper can be easily modified to accommodate the modified HJ-distance case. Importantly, under misspecified models, the misspecification-robust t-statistic will still have a standard normal limiting distribution.

We consider the same four linear asset pricing models analyzed in Table V of Lustig and Verdelhan (2007), and estimate them using the sample modified HJ-distance.[2] The first model considered is the consumption CAPM (CCAPM) with SDF given by

$$y_t^{CCAPM}(\gamma) = 1 - \gamma_1(c_{nd,t} - E[c_{nd,t}]),$$

where c_{nd} is the growth rate in real per capita nondurable consumption. The second model (DCAPM) explicitly breaks down consumption into consumption of nondurables and consumption of durables (c_d):

$$y_t^{DCAPM}(\gamma) = 1 - \gamma_1(c_{nd,t} - E[c_{nd,t}]) - \gamma_2(c_{d,t} - E[c_{d,t}]).$$

[1] See Lustig and Verdelhan (2007) for a detailed description of the data.

[2] Since Lustig and Verdelhan (2007) estimate the models using the ordinary least squares two-pass cross-sectional regression methodology, our results are not directly comparable with theirs.

The third model (EZ-CCAPM)

$$y_t^{EZ-CCAPM}(\gamma) = 1 - \gamma_1(c_{nd,t} - E[c_{nd,t}]) - \gamma_2(vw_t - E[vw_t])$$

is a linearized version of the model of Epstein and Zin (1989) with nondurable consumption and the market return (vw) as risk factors. The last model (D-CCAPM)

$$y_t^{D-CCAPM}(\gamma) = 1 - \gamma_1(c_{nd,t} - E[c_{nd,t}]) - \gamma_2(c_{d,t} - E[c_{d,t}]) - \gamma_3(vw_t - E[vw_t]),$$

is a linearized version of the durable consumption CAPM of Yogo (2006).

Table A.1 about here

Panel A of Table A.1 shows that for all risk factors, we cannot reject the null of $E[x_t(f_{it} - E[f_{it}])] = 0_N$. Similarly, for all four models, we cannot reject the null of deficient rank at any conventional significance level.[3] At the same time, none of the models is rejected by the specification test based on the modified HJ-distance at the 5% confidence level. However, as our theoretical analysis suggests, the outcome of the specification test based on the HJ-distance should be interpreted with caution when useless factors are included in the model. In the useless factor case, the HJ-distance test will be inconsistent under the alternative and may have very low power in rejecting misspecified models.

In addition, Panel D clearly shows that no factor survives the model selection procedure based on misspecification-robust t-tests. Even the market factor does not appear to be priced, probably given its small (9%) average absolute correlation with the excess returns on the eight currency portfolios. Consistent with Burnside (2011), our analysis based on the modified HJ-distance shows that there is not enough statistical evidence for us to conclude that consumption growth risk prices the cross-section of carry trade portfolio returns.

B Momentum Portfolios and Industrial Production Growth

Liu and Zhang (2008) claim that the growth rate of industrial production explains the cross-sectional variation in momentum portfolios, yet the average absolute correlation between the momentum portfolio returns and the growth rate of industrial production is only about 2%.

[3]In this context, for a model to be identified, we need the covariance matrix between the excess returns on the test assets and the risk factors to be of full column rank.

The return data are the monthly excess returns on ten momentum, ten size and ten book-to-market portfolios from January 1960 until December 2004.[4]

We consider the same four linear asset pricing models analyzed in Table V of Liu and Zhang (2008), and estimate them using the sample modified HJ-distance.[5] The first model considered is a one-factor model (MP1) with the growth rate of industrial production, mp, as the only factor. Its SDF is given by

$$y_t^{MP1}(\gamma) = 1 - \gamma_1(c_{mp,t} - E[c_{mp,t}]).$$

The second model (FF3) is the usual three-factor model of Fama and French (1993) with SDF given by

$$y_t^{FF3}(\gamma) = 1 - \gamma_1(vw_t - E[vw_t]) - \gamma_2(smb_t - E[smb_t]) - \gamma_3(hml_t - E[hml_t]),$$

where vw, smb and hml are the market, size, and book-to-market factors of Fama and French (1993), respectively. The third model (MP4) is an augmented FF3 model with mp as the additional factor. Its SDF is given by

$$y_t^{MP4}(\gamma) = 1 - \gamma_1(vw_t - E[vw_t]) - \gamma_2(smb_t - E[smb_t]) - \gamma_3(hml_t - E[hml_t]) - \gamma_4(mp_t - E[mp_t]).$$

The last model is the intertemporal CAPM-type model proposed by Chen, Roll and Ross (CRR, 1986):

$$\begin{aligned} y_t^{CRR}(\gamma) &= 1 - \gamma_1(ui_t - E[ui_t]) - \gamma_2(dei_t - E[dei_t]) - \gamma_3(uts_t - E[uts_t]) \\ &\quad -\gamma_4(upr_t - E[upr_t]) - \gamma_5(mp_t - E[mp_t]), \end{aligned}$$

where ui, dei, upr, and uts denote the unexpected inflation, the change in expected inflation, the term premium, and the default premium factors, respectively.

Table B.1 about here

Table B.1 shows that all four models are strongly rejected by the data. In addition, we cannot reject the null of deficient rank for MP4 and CRR at the 5% nominal level and only FF3 appears to be identified at any conventional nominal level. The results of the specification and rank restriction tests clearly point to the need for robust statistical methods.

[4]We refer to Liu and Zhang (2008) for a detailed description of the data.

[5]Since Liu and Zhang (2008) estimate the models using the ordinary least squares two-pass cross-sectional regression methodology, our results are not directly comparable with theirs.

Consistent with the other empirical illustrations, all the non-traded factors do not survive the model selection procedure based on misspecification-robust t-tests. The only evidence of pricing comes from the market factor and the book-to-market factor of Fama and French (1993). In summary, there is not enough statistical evidence for us to conclude that industrial production risk prices the cross-section of momentum portfolio returns.

C The Stock-Watson Factors

The paper considers several models with macroeconomic variables as risk factors. It is possible that these individual factors do not capture adequately the risk incorporated in all of the macroeconomic data that is available to market participants. One approach to extract parsimoniously the common variation in macroeconomic variables is the factor analysis advanced by Stock and Watson (2002a, 2002b). See also Ludvigson and Ng (2007, 2009) for a similar approach in the analysis of stock and bond risk premia. In this section, we follow Stock and Watson (2005) and construct three orthogonal factors that summarize the dynamics of 127 macroeconomic time series for the period February 1959 – July 2007. We use the same variables and transformations as described in Stock and Watson (2005) with the exception of four series (houses authorized by building permits for Northeast (BP: NE), Midwest (BP: MW), South (BP: South) and West (BP: West)) for which there are missing observations for the beginning of the period.[6]

Let x_{it} ($i = 1, ..., N$, $t = 1, ..., T$) denote the i-th observed series at time t, where N is the total number of variables and T is the number of time series observations. When the cross-sectional dimension N is large, the use of all these variables as risk factors is impractical and even infeasible if $N \geq T$. Instead, suppose that x_{it} admits an approximate factor structure of the form

$$\begin{aligned} x_{it} &= \lambda_i' f_t + e_{it} \\ A(L) f_t &= u_t, \end{aligned} \tag{C.1}$$

where f_t is a $K \times 1$ vector of latent common factors, λ_i is a $K \times 1$ vector of latent factor loadings, e_{it} is a vector of idiosyncratic errors, $A(L)$ is a possibly infinite dimensional lag polynomial, and u_t is a vector of *iid* errors with mean zero and a constant variance-covariance matrix. The idiosyn-

[6] We would like to thank Toni Braun for suggesting this to us and Sydney Ludvigson for making the data available on her website.

cratic shocks are assumed to be uncorrelated with the factors at all leads and lags although serial correlation, heteroskedasticity, and a limited amount of cross-correlation is permitted (Stock and Watson, 2002a). The selection of the number of factors in the approximate factor model (C.1) is considered in Bai and Ng (2002).

Let X denote the stacked $T \times N$ data matrix with its t-th row given by $x_t' = [x_{1t}, x_{2t}, ..., x_{Nt}]$ and $F = [f_1 \ f_2 \ ... \ f_K]$ be the $T \times K$ matrix of common factors. Provided that $N, T \to \infty$, the latent factors and factor loadings can be estimated by the method of principal components by minimizing the objective function $(NT)^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} (x_{it} - \lambda_i' f_t)^2$ subject to the identifying restriction $F'F/T = I_K$. Concentrating out $[\lambda_1', ..., \lambda_N']'$, the estimate of the factor matrix F, $\hat{F}$, is obtained by maximizing $\mathrm{tr}(F'(XX')F)$ and $\hat{F}$ is a matrix of $\sqrt{T}$ times the K eigenvectors corresponding to the K largest eigenvalues of the matrix XX'. The optimal number of factors can be determined by the panel information criterion proposed by Bai and Ng (2002). While Stock and Watson (2005) and Ludvigson and Ng (2009) estimate the optimal number of factors to be 7 and 8, respectively, we restrict our attention to the first three principal components given their clear economic interpretation. More specifically, looking at the marginal R^2 from a regression of each individual series on each estimated factor, Ludvigson and Ng (2009) interpret the first estimated factor as a real activity factor, the second estimated factor as a financial factor that loads most heavily on interest rate variables, and the third estimated factor as an inflation factor. Also, note that while the factors $\hat{f}_t = [\hat{f}_{1,t}, \hat{f}_{2,t}, \hat{f}_{3,t}]'$ $(t = 1, ..., T)$ contain an estimation error, no adjustments to the standard errors of the risk premium estimates are required provided that $\sqrt{T}/N \to 0$ (Bai and Ng, 2006).

We then consider the following SDF:

$$y_t^{SW3}(\gamma) = \gamma_0 + \gamma_1 \hat{f}_{1,t} + \gamma_2 \hat{f}_{2,t} + \gamma_3 \hat{f}_{3,t}.$$

The sample period and the returns on the test assets are the same as the ones used in the monthly analysis in the paper. Table C.1 presents our results.

Table C.1 about here

The rank tests for individual factors indicate that the first (real activity) and the third (inflation) factors can be reasonably considered as useless (see Panel A). This message is further reinforced by the rank restriction test for SW3 in Panel B. Given that the second factor appears to be the only statistically significant factor (even when the misspecification-robust standard error is used), the model is re-estimated after the first and the third factors are dropped from the model. Interestingly, for the model that contains only the second estimated factor (SW1), the rank test rejects the null of reduced rank and the interest rate factor is highly statistically significant. This finding is again in line with our theoretical results and the other empirical applications.

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Table A.1
Estimation and Testing Results for the Linear Models Considered by Lustig and Verdelhan (2007)

The table presents the estimation and testing results for the four linear asset pricing models considered by Lustig and Verdelhan (2007). The models considered are the consumption CAPM (CCAPM), a CCAPM specification (DCAPM) with consumption of durables and nondurables as risk factors, a linearized version of the model of Epstein and Zin (EZ-CCAPM, 1989) with consumption of nondurables and the market return as risk factors, and the durable consumption CAPM (D-CCAPM) of Yogo (2006). The models are estimated using annual excess returns on eight currency portfolios. The data are from 1953 until 2002. Panel A reports a Wald test ($\mathcal{W}$) and its p-value (p-val) of the null that $E[x_t(f_{it} - E[f_{it}])] = 0_N$. This Wald test has a χ^2_N limiting distribution. In Panel B, we report the sample HJ-distance ($\hat{\delta}$) and the rank restriction test ($\mathcal{W}^*$) with the corresponding p-values (p-val) for each model. When $K = 1$, the reported $\mathcal{W}^*$ corresponds to the $\mathcal{W}$ test described in Panel A. The t-tests under correct model specification and the model misspecification-robust t-tests are in Panels C and D, respectively. Each t-test is for the test of the null hypothesis that the coefficient associated with a given risk factor is equal to zero.

Panel A: Wald Tests for Individual Factors

Test	vw	c_{nd}	c_d
$\mathcal{W}$	7.2	9.2	6.5
p-val	0.518	0.324	0.587

Panel B: HJ-Distance and Rank Tests

Models	$\hat{\delta}$	p-val	$\mathcal{W}^*$	p-val
CCAPM	0.754	0.179	9.2	0.324
DCAPM	0.657	0.165	1.5	0.982
EZ-CCAPM	0.751	0.092	2.8	0.899
D-CCAPM	0.649	0.122	1.5	0.962

Panel C: t-tests Using Standard Errors Under Correct Model Specification

Models	vw	c_{nd}	c_d
CCAPM		1.90	
DCAPM		0.41	1.47
EZ-CCAPM	−0.25	2.31	
D-CCAPM	0.50	0.15	1.63

Panel D: t-tests Using Model Misspecification-Robust Standard Errors

Models	vw	c_{nd}	c_d
CCAPM		1.24	
DCAPM		0.26	0.96
EZ-CCAPM	−0.20	1.46	
D-CCAPM	0.33	0.07	0.95

Table B.1
Estimation and Testing Results for the Linear Models Considered by Liu and Zhang (2008)

The table presents the estimation and testing results for the four linear asset pricing specifications considered by Liu and Zhang (2008). The models considered are the one-factor model (MP1) with the growth rate of industrial production, *mp*, as the only risk factor, the three-factor model of Fama and French (FF3, 1993), the FF3 model augmented with *mp* (MP4), and the intertemporal CAPM-type model of Chen, Roll, and Ross (CRR, 1986). The models are estimated using monthly excess returns on ten size, ten book-to-market and ten momentum portfolios. The data are from 1960:1 until 2004:12. Panel A reports a Wald test ($\mathcal{W}$) and its p-value (p-val) of the null that $E[x_t(f_{it} - E[f_{it}])] = 0_N$. This Wald test has a χ^2_N limiting distribution. In Panel B, we report the sample HJ-distance ($\hat{\delta}$) and the rank restriction test ($\mathcal{W}^*$) with the corresponding p-values (p-val) for each model. When $K = 1$, the reported $\mathcal{W}^*$ corresponds to the $\mathcal{W}$ test described in Panel A. The t-tests under correct model specification and the model misspecification-robust t-tests are in Panels C and D, respectively. Each t-test is for the test of the null hypothesis that the coefficient associated with a given risk factor is equal to zero.

Panel A: Wald Tests for Individual Factors

Test	*vw*	*smb*	*hml*	*mp*	*ui*	*dei*	*uts*	*upr*
$\mathcal{W}$	246.4	222.8	247.3	48.1	35.3	43.5	48.4	44.4
p-val	0.000	0.000	0.000	0.020	0.231	0.052	0.018	0.044

Panel B: HJ-Distance and Rank Tests

Models	$\hat{\delta}$	p-val	$\mathcal{W}^*$	p-val
MP1	0.449	0.000	48.1	0.020
FF3	0.405	0.000	119.0	0.000
MP4	0.399	0.000	39.1	0.063
CRR	0.411	0.003	5.0	1.000

Panel C: t-tests Using Standard Errors Under Correct Model Specification

Models	*vw*	*smb*	*hml*	*mp*	*ui*	*dei*	*uts*	*upr*
MP1				1.87				
FF3	3.37	1.55	4.19					
MP4	3.10	1.39	3.74	1.39				
CRR				1.51	−2.25	2.24	0.91	1.25

Panel D: t-tests Using Model Misspecification-Robust Standard Errors

Models	*vw*	*smb*	*hml*	*mp*	*ui*	*dei*	*uts*	*upr*
MP1				1.10				
FF3	3.39	1.56	4.18					
MP4	3.11	1.40	3.71	0.89				
CRR				1.04	−1.15	1.00	0.45	0.59

Table C.1
Estimation and Testing Results for the Models with the Factors of Stock and Watson

The table presents the estimation and testing results for the asset pricing specifications with the factors of Stock and Watson (2002a, 2002b) constructed from 127 macroeconomic time series. SW3 denotes the model that includes the three estimated factors and SW1 denotes the model that includes only the second estimated factor. The models are estimated using monthly gross returns on the 25 Fama-French size and book-to-market ranked portfolios and the one-month T-bill. The data are from 1959:2 until 2007:7. Panel A reports the rank restriction test ($\mathcal{W}^*$) and its p-value (p-val) of the null that $E[x_t(1,\ f_{it})]$ has a column rank of one. In Panel B, we report the sample HJ-distance ($\hat{\delta}$) and the rank restriction test ($\mathcal{W}^*$) with the corresponding p-values (p-val) for each model. The t-tests under correct model specification and the model misspecification-robust t-tests are in Panels C and D, respectively. Each t-test is for the test of the null hypothesis that the coefficient associated with a given risk factor is equal to zero.

Panel A: Rank Tests for Individual Factors

Test	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$
$\mathcal{W}^*$	35.7	56.8	23.2
p-val	0.076	0.000	0.564

Panel B: HJ-Distance and Rank Tests

Models	$\hat{\delta}$	p-val	$\mathcal{W}^*$	p-val
SW3	0.360	0.002	18.2	0.749
SW1	0.364	0.001	56.8	0.000

Panel C: t-tests Using Standard Errors Under Correct Model Specification

Models	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$
SW3	0.83	−3.81	−0.31
SW1		−4.40	

Panel D: t-tests Using Model Misspecification-Robust Standard Errors

Models	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$
SW3	0.43	−2.59	−0.16
SW1		−3.69	